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ADDRESS AT THE ANNUAL CONVENTION AT THE HOTEL KAATERSKILL, NEW YORK, JULY 2, 1887,

BY WILLIAM E. WORTHEN, President Am. Soc. C.E.

I was much gratified by my election as President of the American Society of Civil Engineers, a numerous body of men who fully represent the profession, and with whom I have been for a long time connected. In assuming the office I undertook the duties of the position, which, however honorable, are not entirely without responsibility. I now proceed to fulfill one of the requirements by delivering an annual address, which is supposed to be a *resumé* of what was done last year and the progress of the work of engineers, with some impressions of my own experience and my opinion with regard to the future.

I have collected sundry statistics from newspapers, magazines, etc., and although I have not credited them to the different parties, I yet follow the rule of the society in their publications, in not assuming the responsibility for the facts and opinions advanced, nor shall I back my opinions against facts if they are hereafter found in opposition thereto.

There is little of the catastrophic in engineering science. It is essentially an evolved science; the works and facts from which it is

evolved have been the accumulation of centuries; shrewd guesses or judgments worked out; a copying of examples; changing construction according to the teaching of experience, and application of varied material dependent upon location and requirements. Without going back into remote history, when the term engineer was not known, although there were wonderful examples of engineering construction, let us consider the growth of the science up to its present standpoint. Who built the first bridge? The change from wooden to iron structures is recent, yet how full is the record of steps. Who conceived the first railway? Its growth from the strap railway is wholly within our recollection. How many have contributed to make the turbine the wonderfully efficient and economic water-motor it now is? Who first thought of the application of steam to the purposes of power? How many worked at the construction and improvement of boilers and steam engines? We can pick out parts that were an essential improvement, and give the names of the men who first designed them, but the great body of the machine is made up of parts which have been constructed by mechanics without name, and not only the parts, but the machines to make the parts uniform and interchangeable.

Many an invention conceived before its time could be unearthed from patent records; either because it was not then needed, or the tools which made it economically and practicable were not to be had. How much development of industry has been given by the lathe, planer, and shaping machines? Seldom, if ever, in the lapse of a single year can we record any striking novelty which can be set down as a step in progress. It takes years to complete and practically test. Like the statue of "Liberty Enlightening the World," which is one of the accomplished facts of last year, it took many years to design, construct, and raise it. It is unfortunate, however, that in illuminating the world, no provisions were made for illuminating the statue itself.

The name Engineer is of quite recent date, and is very mixed in its definition; the Architect preceded it. Architecture was always considered among the ancients as one of the liberal arts, but the name of architect was not given till a few centuries ago to artists charged with the construction of buildings. The man of the profession before the architect was the master of the works, a practical builder; and work comprehended the entire building and its furniture. It is probable that the religious houses, which contained all that there were of men of

letters, savants, and scholars on the Continent, furnished not only the men who designed and directed the monastic constructions, but also the civil ones, and perhaps military. The master of works became the head of a corps of workmen which made a part of the free corporations which regulated wages and hours of labor, and defined rights and duties. Once outside of the cloister, work went on with great rapidity. The master workman was the acknowledged head; his name was connected with his work, and he was buried within the most prominent church he constructed. But the necessities of variation in designs and details soon brought in the artist and the architect, and something beside the construction of buildings was necessary.

In 1560–1641, Sully, Grand Master of Artillery and Superintendent of Finances of Henry IV of France, was the first that appreciated the service that could be rendered by a body of geometricians and mathematicians, and he organized a *Corps de Génie*, which took the title of "*Ingénieurs Ordinaires du Roi.*" This organization remained unchanged until the time of Colbert and Vauban, the latter the "*vrai père de Génie Civil et Militaire.*" L'Ecole Mezieres, established in 1748, was the first institution devoted to the development of the sciences. This was the period when the expression the "*Corps de Génie*" was introduced into the army. In England, the distinction between engineers became that of civil and military. It was evident in the discussion of the paper of Mr. William Metcalf, M. Am. Soc. C. E., on Steel, before this society, that in the application of this material to the construction of guns and to civil purposes, no distinctions were drawn between the civil and the military, and the graduates from West Point are able civil engineers and fulfill that duty under the direction of the War Department. In general, the term engineer is supposed to come from engine, and the term engineer embraces not only the man who designs and constructs it, but the man who runs it. I suggest that there should be some distinction; that engineers should be such as are eligible to our society, and that the men who run and take charge of engines or machines should be called *enginers*. I want to reclaim, perhaps, a little more of "*le génie*" for the profession than for the mechanical artisan; for skill, experience, and industry may make a very good workman, but to this must be added more general and extended information in the application of materials to necessities, mathematics to calculate, and drawings to express; and, more than this,

a sentiment of purpose, a feeling for the work. A true engineer is as much born as a poet. Could we divide the application of the term into engineers and enginers it would be distinctive, and I would drop the term civil as embracing too large a profession, and adopt prefixes as military, hydraulic, railway, mechanical, steam, electric, and other divisions, when these are adopted as specialties. With regard to military engineering, I know of no particular progress of recent date. The War Department has undertaken to clear out Gedney's Channel, in New York Harbor, by pump dredging, and it is satisfactory to report that the work last year retained its depth without filling in, and a new contract has been entered into at about half the price of last year and is successfully progressing.

HYDRAULIC ENGINEERING.—CONSTRUCTIONS FOR THE USE OF WATER FOR POWER, CITY AND TOWN SUPPLIES, HYDRAULIC MINING AND IRRIGATION.

In the census of 1880 will be found tabulated accounts of the water-power of the United States, developed and undeveloped. Attention is called to the low percentage of increase of water power in comparison with that of steam:

In 1870, Water power,	1,130,431 H.P.	Steam,	1,315,711 H.P.
In 1880, "	1,225,379 "	"	2,185,418 "
Percentage of increase,	8.40 "	"	79.77

It will be seen that the percentage of increase of steam is very much in excess of that of water, but the latter still is not an inconsiderable amount, 95,000 horse power, nearly equal to the power of the City of Lowell, per year. A convenience of access to business and labor centers has tendered very largely to the increase of steam power. And although there are immense water powers yet undeveloped, and the cost of steam power is largely in excess of that of water power, yet position and its relations have decided in favor of steam. And where large manufactories have been commenced on water power. and where the water power has become exhausted, or very irregular, the business has been extended by the use of steam, rather than by a diversion of it to a new place for water. In 1844 I assisted Mr. U. A. Boyden in conducting his experiments on the first Fourneyron turbines at the Appleton Mills, Lowell, Mass. This wheel gave 76 per cent. useful effect from the water, whilst some breast-wheels at Lowell, which Mr. James B.

Francis, Past President Am. Soc. C. E., and myself tested some time before, gave from 60 to 66. Mr. Boyden afterwards constructed many other turbines throughout the country, introducing many improvements in design and construction. One at Lawrence gave over 90 per cent. of effect. Boyden came of a very remarkable family, and had, above all men that I ever knew, that desirable genius, the genius of industry. He filled in all his time with scientific work and research. His relaxations consisted almost entirely of changes of subject. His improvements in water-wheels increased the water power at Lowell about one-fourth over what was obtained by the former wheels, and gave him the means of pursuing other investigations of light, heat, sound, etc. His mode of experimenting was that now followed by Mr. Francis at Lowell and Mr. Mills at Lawrence. With the impulse given by Mr. Boyden, water-wheel shops were established in different parts of the country. The wheels were tested and improvements made, until now water-wheel construction has passed out of the hands of the engineers into those of manufacturers, who furnish wheels ready made, simple, efficient and strong, guaranteed to give 80 per cent. effective power.

Water Supply of Towns.

There is a very large increase in the number of town and cities supplied and additions to work already constructed. The great improvement in the matter of aqueducts has been in the application of wrought-iron for cast-iron mains, and also to take the place of masonry and tunneling. Large wrought-iron pipes for conducting water to turbines have been in use from the time of the introduction of the Boyden turbine wheel, but it has been left for California to largely extend its use for the conducting of water for hydraulic mining, irrigation and town supply, with a boldness of design and execution which has astonished the engineer. The method of hydraulic mining was introduced or invented in California in 1852. The supply of water for these jets at first was conducted through hose made of heavy duck cotton cloth, which was strengthened by outer nettings of cordage when the pressure was large. In 1853 an ingenious miner laid in his mine a line of pipe consisting of ordinary stove-pipe, made of very thin sheet-iron, lightly fastened together with cold rivets, with the joints united stove fashion. This pipe was five or six inches in diameter. As hydraulic mining increased in magnitude the sizes of these supply pipes also increased, the

diameter for main lines for a large mine being from 22 to 30 inches. These pipes, as a rule, are made at the mine. The iron is made from .055 to .134 inches (No. 16 to 10.Birmingham gauge) in thickness, with a double row of cold rivets for the longitudinal seam when the pressure is to be large. As a protection against rust, each joint is immersed for several minutes in a bath of boiling asphaltum and tar. They are now used for permanent conduits both for conducting water to mining districts, across deep mountain gorges, and also for the supply of cities. San Francisco, a place of some 300,000 inhabitants, receives its water through two lines of such pipes, and a third pipe, many miles in length and of large diameter, is now being laid for an additional supply. Within the last year one of the large shops in San Francisco has increased and improved its plant, so that they are now able to turn out weekly a mile of pipe 44 inches in diameter. Of the quantity of water necessary per inhabitant there is considerable difference of opinion, but there is no doubt in the general opinion, that in the present use of water there is an extensive waste.

Of the quality of water for a town supply there is a great difference of opinion among chemists and scientists. M. Gerardin (*Rapport sur l'Alteration, la Corruption et l'Assainissement des Rivieres*) says "the distinction between healthy waters and polluted ones cannot be detected by color, smell, taste, or chemical analysis. Water can be very much polluted without giving any smell. Such are the waters of paper-mills, starch and sugar-works at the leaving of the works. Many natural waters which are prescribed in the case of sickness are very offensive in smell. Chemical analysis does not sufficiently explain the alteration of waters. If a perfectly healthy water be inclosed some time in a bottle, well-corked, its elementary analysis always manifests the same results, and, notwithstanding this, it has undergone radical change and has lost all its good qualities."

Chaptal says "that those who make a chemical examination of waters only analyze its dead body." "Water is healthy when animals and vegetables of a superior organism can live in it; on the contrary, a water is affected when animals die, and it can only nourish infusoria. There is no better means of establishing the character of water than by determining if in this water fish and aquatic plants can live. If fish die there and plants perish there, the character is certainly polluted, and the water should not be used for domestic purposes. On the contrary,

if fish and aquatic plants can live in it the water can be considered good." There is an old French proverb, "*Il n'est pire eau que l'eau qui dort*" (There is no worse water than that which sleeps) which conveys the general impression with regard to the sources of supply. A running stream supplies the best water, and when reservoirs are necessary, the one from which the water is drawn directly should be as small as possible, and the water from superior reservoirs should flow down into it, so that the water may be well aerated in its course. Organisms are not dangerous because microscopic, and they are not, as a rule, dangerous. "It is only," as an eminent scientist observed, "when germs of disease are introduced from the outside, that the waters become dangerous," and organisms actually contribute largely to the purification of the water. If, by the doctrine that water once polluted is always polluted, it is to be understood that these pollutions are accumulations from century to century, then, for the preservation of our species, our own organisms must be changed to adapt us to the circumstances, and in our new evolution we should not be suited to the air and water of the Garden of Eden and its suburbs. M. Gerardin, in the work above quoted, speaking of *algæ*, says they are endowed with a very active respiration. "The oxygen which they accumulate decomposes rapidly organic matter, and they contribute largely to the purification of waters polluted by organic matters in decomposition. From the biological examination of the water of various streams that are polluted by paper-mills, and sugar, starch, and glue-works, as it leaves the works, and at different stages of the stream below, he finds that *cryptograms* and *algæ* first begin to appear, and thus up through the variety of forms to the higher and higher organisms, until fish appear in their waters and plants along their shore." Experiments in aquaria confirm this conclusion. I would refer to the report on pisiculture of James B. Ferguson, Commissioner of the United States at the Paris Universal Exposition of 1878, on the value of the *algæ*. He says: "Oxygen in its native state, resulting from the respiration of vegetable, and especially from microscopic *algæ*, is very soluble in water. Atmospheric air introduced mechanically into water is not dissolved there, or if so in very small quantities." He quotes also from the Superintendent of the Aquarium his experiments on the aeration of waters supplied, as follows:

"The water of the Vanne, taken almost from its source, contains consequently little air and no organic germs; hence, although very

useful for the cleansing and purification of the basins, it was not suitable to favor the rapid development of a cryptogamous vegetation upon the sides of the tanks. I substituted, therefore, the waters of the Seine, the color of which was, at that time, green on account of the abundance of microscopic germs, sporules of *algæ*, *conferva*, and, in fact, all those vegetable aquatics—for the most part microscopic—the presence of which is requisite for the continuous oxygenation of water intended to receive fish. In truth, upon water, as well as upon air, has been imposed that great law of nature which counterbalances and equalizes the animal and the vegetable kingdoms, making the one the indispensable auxiliary of the other.

"Fish, as well as animals on land, absorb oxygen by respiration, and give out carbonic acid, the quantity of which, always increasing in the place which they inhabit, would not fail to produce asphyxia if the aquatic vegetable were not present to produce a chemically inverse effect. Under the influence of solar radiation the green immersed particles absorb carbonic acid, decompose it, assimilate the carbon and exhale the oxygen, which appears upon their surface in innumerable small bubbles, supplying the fish with the respiratory element, renewed and endowed with a vital action so much the more efficacious because just brought to life, and possessing a solubility much greater than that of oxygen drawn from the atmosphere.

"Perhaps there is even a formation of bioxide of hydrogen, which would explain this coefficient of solubility as being so much greater than that admitted by chemists. However that may be, for me the fact is now incontestable that to establish in an aquarium a hygienic medium suitable for the life of aquatic animals, an abundant vegetation must there be developed, which should frequently undergo the action of solar radiation."

Most successful aquaria are constructed with what may be termed closed circuits, not drawing water from an exterior supply, except to make up waste. Water is used over again and again by aerating it in its circulation. Mr. C. B. Brush, M. Am. Soc. C. E., has written a paper which will be read before the Society, of experiments in forcing air into water by means of air-pump pressure at the Hackensack works, as he claims, with success.

Whether the cucumber taste of the Croton water be due to the *spongilla lacustrina* or to *nostochoniæ*, it seems to me to be rather due to the

want of circulation, and the decease and decay of some organism by which the balance of aquarial life has not been properly kept up. The pabulum has failed, and perhaps it might be remedial to introduce a little sewage, as it has been shown by analysis that the sewage at Troy improves the water of the Hudson, and that the water supply of Albany is better than that above. The *algæ* called frogs' spittle, *Batrachosperma moniliforma* (I might here remark, the smaller the organism the bigger the name), will always be found in the purest spring water. I am pleased to see them, and feel more sure of the purity of water by the presence of such common organisms than by their absence, understanding that within the microcosm there is a class of organism which depends on the usual food supplied by nature. Disturbance means disease, if not death. *Post hoc* is not always *propter hoc;* don't at once ascribe disease to the presence of an organism; and when you see crows or buzzards hovering around the carcass of a horse, don't come to the conclusion that they have killed a valuable animal and shoot the crow or buzzard.

In the early times of gold-mining in California, hydraulic mining was the rule; but with the increase of value for agricultural purposes, the disturbance of rivers, and the covering of land with *debris*, this has been found very objectionable, and water has taken a value for irrigation. But with irrigation developments the necessity of defining water-rights has become indispensable, and a law was passed creating the office of State Engineer, and duties were assigned opening a large field of inquiry over an immense territory. The field of work is in the nature of a physical survey of the State, combined with certain industrial, social, and other inquiries of a broad scope, and necessary to be prosecuted over a wide field.

The first part of the report has now come out, and it is for sale by the State of California. "It is introductory to the study of problems of irrigation, and is composed of a series of studies of irrigation development in the three countries of France, Italy, and Spain, from whose experience we propose to learn something for our own immediate purposes." We have referred to this report not only for its value in matters treated, but because like necessity exists in all our States to define the rights of water-users, whether for power or town supply; the drainage of land for agricultural purposes and sometimes irrigation; and in the discharge of polluted matter from streams. In whom does the title to

water lie? Is it the first user? When the large city takes the entire flow of contiguous rivers and streams, whence is the fast increasing suburban population to be supplied, and once taken has any town or city the right to deprive others of this by waste? In 1883 an act was introduced into the New Jersey Legislature to provide for the appointment of commissioners to determine on plans for the storage of any of the waters of that State for the purpose of furnishing to cities and towns a joint water supply. With regard to this, commissioners were appointed and a small amount of money was provided for expenses. The report was made, but no further action was taken. The commission is still in existence, but without power or money. In 1886 an act was passed by the Commonwealth of Massachusetts, requiring from the Board of Health a report on the condition and means for protecting inland waters. A report has been made under this act. This year a new bill was introduced to establish a Board of Water Commissioners, to provide for the establishment of water companies and the general supervision of drainage connected with inland waters. Such boards are necessary in every State, not only to define the uses of water and the removal of waste within the limits of the State, but also there are interstate rights, and you can readily understand that the majority of such boards should be hydraulic engineers and with such powers as the State can delegate.

Steam Engineering.

In 1857, Mr. Charles W. Copeland and myself made the first experiments on pumping engines at Belleville, N. J., and Hartford, Conn. (Published in the *Brooklyn Water Works*, and issued by the City of Brooklyn). It may be remarked that the evaporation of the boilers, 10 to 11 pounds of water per pound of coal from water at 100 degrees, will still be considered a very fair evaporation. But the duty per 100 pounds of coal was 55 000 000 to 62 000 000 on the Hartford engine and 68 000 000 to 73 000 000 on the Cornish engine at Belleville, now very much below the duty of first-class pumping engines. From the above experiments, Mr. James P. Kirkwood, Past President Am. Soc. C. E., felt authorized to require by contract for the first Brooklyn engine a duty of 60 000 000 pounds feet per 100 pounds of coal. It is not now unusual to require 100 000 000 pounds feet, or one-horse power for a little less than two pounds of coal per hour, and this taken on

the water end of the machine. First-class pumping engines are now invariably compounded, and compounds have also been largely introduced in stationary engines and with largely economic results; of their exact comparative economy we need more tests.

When steam was first introduced for the heating of cotton-mills, a boiler was placed at each end of the mill, with automatic water-feed by means of balance stone float. Later the boilers were concentrated within one house and the steam distributed through ranges of immense mills. The original steam-heating pipes were cast-iron, about 3 to 4 inches in diameter, and suspended from the ceiling; later, with the introduction of wrought-iron pipe, the pipe was carried around the walls of the mill near the floor, but at present the wrought-iron steam-pipes are suspended from the ceiling. In mills where large quantities of steam are used for other purposes than power, it is common to pass the steam through a steam engine for power, and then use the exhaust for drying, boiling, and heating. In 1866 I made a report to the Merrimack Manufacturing Company, of Lowell, on the economy of using steam first for power at the cotton-mills and afterwards the exhaust for their print works for the purposes of heating, boiling, etc., and reported on the advisability of passing the steam through engines of 1 200 horse power, and using the exhaust steam afterwards in the print works, with a back pressure of about six pounds, and that the amount of coal properly chargeable to power in this case would not exceed one pound of coal per horse power per hour. It was a success, and of its present extent the superintendent of the works writes me as follows: "We have only one pair (34 by 72) condensing engines on our premises; all the others (something over 100 cylinders) exhaust into a system of mains, from which we draw all the heat we require. What we do not require for heat we take into the condensing cylinders before mentioned, and reconvert into power." Some time since there was a boom in the steam-heating of towns and cities from central stations, and many plants were constructed many of which have gone out. There are still two in New York City, one of moderate size, comparing fairly well in extent with that of a large cotton-mill plant; the other is very large and is still increasing its distribution. This result seems to argue the survival of the fittest.

Railroads.

There has been substantial progress in the extension of railroads, in

the construction of new roads, and doubling and quadrupling the old ones. The estimation of construction for the year is 10 000 miles. The combined locomotive has been tested in Europe, but not sufficiently to afford a fair test of its practical value, and has not been put in operation here. The public has been greatly exercised by numerous accidents in the destruction of cars and passengers by fire, and our legislators have called for a safer means of heating and lighting than at present in use; in New York one year is given for the change. The two great questions of automatic freight-train brakes and automatic freight-car couplers have made great progress during the past year. The freight-brake tests which were made at Burlington, Iowa, in July of last year, have been supplemented by a second series this year, the two series together being by far the most complete tests of the kind which have ever been made. The results demonstrated that only air brakes are adapted to the requirements. The prominent part of this year's tests was that taken by electricity as an actuating agent for applying the brakes, with results showing great benefit from its use, both in quickness of stop and in avoiding the dangerous shocks which result from quick stops with long trains. The electric application being absolutely simultaneous throughout the train, no shock whatever results. The occasional failures of the electric apparatus during the test indicated that further improvements are still needed, but as electricity simply supplements the air, which acts as usual in case of an electric failure, the probability seems strong that such electric application will be a feature of the train-brake of the future, while a percentage of efficiency was shown in this year's test, much greater than was ever shown before. As a safeguard for human life, rapid progress has been made toward the introduction of automatic freight-car couplers. The Burlington brake tests of this year and last show that it was on the whole decidedly beneficial to eliminate loose slack between trains which has tended to exclude couplings of the link and pin type, and the final step has now been taken by the formal adoption by the Master Car Builders' Association of the Janney hook coupler and others of the same type coupling with it as standards of the association, by a vote of 33 to 14.

Street Railways.

The entire service of street railways was for many years by the means

of horses, and we quote as to the cost of working the following returns taken from the sixteenth annual report of the Massachusetts Board of Railroad Commissioners: Highland, Lynn and Boston, Metropolitan, Middlesex, and South Boston; number of horses owned, 6 158; number of miles run, 11 287 196; number of passengers carried, 68 196 776; average number of passengers carried per round trip, 43.4.

From the above it appears that the stable average daily distance traveled by the above horses equals 10.04 miles.

The average number of passengers per round trip being 43.4, per single trip equals 21.7. Averaging them at 140 pounds, equals 3 038 pounds. Add weight of car, 4 800 pounds, equals 7 838 pounds.

With the great extension of street roads there have been found great inconveniences in this horse service, not only in the cost, but in the location of immense stables within populous neighborhoods. The usual traction force has been reckoned to be about $\frac{7}{100}$ per cent. in movement, and from 6 to 7 per cent. in starting on level grades. This will afford us a means of comparison between different forms of motors.

Although the cable was used on the London and Blackwall Railroad for many years and for heavy trains, it was superseded by the locomotive, but it has of late years been revived in this country for the distribution of city travel and with great success.

Chicago has 20½ miles of cable road in operation and 22 under construction; Cincinnati, 8 miles; Kansas City, 4 miles in operation and 30 under construction; New York City, 10½ in operation; Philadelphia, 18 miles; St. Louis, 6 miles; San Francisco, 33½ miles now in use; Melbourne, Victoria, has several miles; London, 3 miles; and four years ago a road was built in New Zealand. The first cable road in San Francisco was opened in 1873. This was the famous "Clay Street Hill Cable Road," where the grades were so steep that property was almost valueless. The cables had no trouble with the grades, 17 feet in 100, where horse power was useless, and the road was an immediate success.

Mr. W. H. Paine, M. Am. Soc. C. E., has furnished me with the following facts in regard to the cable on the Brooklyn Bridge, which he laid: "The practicability of applying cables to the hauling of large heavy passenger cars in trains at considerable speed with shorter headway, greater regularity and less danger from accident than any other method known, is here demonstrated.

"The durability of the cable, and of the grips and dies has been here so great as to remove a most serious obstacle to cable traction.

Speed of cable, 10 miles per hour.
Length of cable, 11 000 feet.
Size of cable, 1¼ inches in diameter.
Weight, 3½ pounds per lineal foot.

"The first cable was taken out last November, after having been in use three years and 41 days, having hauled 1 675 790 cars, 48 108 567 passengers, or 22 196 436 tons. The heaviest cars weigh 16 tons when empty, the heaviest load of passengers about ten tons. The lightest cars weigh 11 tons when empty. Three cars are now hauled in one train. Shortest headway about 1½ minutes. More than 100 000 passengers have been carried in one day. The grips will last for years with few repairs. The dies or lining of the grips last more than four months, sometimes exceeding five months. The speed of the cable, which is 10 miles per hour, could be increased, but without increasing the carrying capacity of the road."

Electric Railways

for street passenger traffic seem now to be a fixed fact. In Europe electric street railways are now carrying at the rate of 3 000 000 annually, and probably a little in excess of that number here, and they are extending. The first built was in Berlin, Germany, 1881, 1½ miles in length. In this country they date from 1885. The following are the places in which they are established: Baltimore, Md.; Los Angeles, Cal.; Port Huron, Mich.; Windsor, Canada; Detroit, Mich.; Appleton, Wis.; Scranton, Pa.; Denver, Colo.; Montgomery, Ala. In all but one of these the dynamo is driven by a steam engine, but in Appleton, Wis., there is a road of 4½ miles where water power is used. You may recollect that an eminent divine once remarked as an evidence of design in the works of the Creator, that great rivers were placed near great cities. It were fortunate for the purpose of lighting and heating if great water-powers were placed in the vicinity of large, populous districts. It has not yet been sufficiently decided how far power can be carried by means of electricity, nor the size of the conductor and the loss in conducting it. With the short roads now in operation it has been said that the percentage of effect on the car is 50 to 55 per cent. that of the steam engine

providing the power. In this respect it is superior to the cable, which is fast superseding horses.

Few of the present engineers recollect the experiment of the old atmospheric railway of Samuda, which ran from Dublin to Kingston, about 1½ miles, successfully for many years; and was later tried in a larger way from London to Croydon and from Paris to St. Germain. It failed and was superseded by the locomotive. It may be that for a moderate distance the atmospheric road may become practicable, but for the present our hopes of a successful motor, especially for city and town roads, are based on electricity. So much attention at least has been directed to this subject, not only with reference to power, but to light and heating, that a new branch of engineering has sprung up and has inaugurated itself into a society like our own as the American Institute of Electrical Engineers. From our knowledge of them and their claims I feel satisfied that they are to be a worthy branch of engineers. In our own country capitalization in electrical districts, telegraphic, telephonic, electric lighting, and electric power, amounts to a sum of at least $350 000 000. There are in America about 700 000 miles of telegraphic wire; last year 32 000 000 conversations were held on the telephone; there are ever 150 000 arc lamps, and incandescents verging on a million; and at least 15 000 electric motors for fans and sewing machines.

Bridges.

The first Howe truss was built across the Connecticut River, at Springfield, Mass., in 1840. It was a wooden bridge with double intersections. Smaller bridges had been built in the same line of road, but this was a very important bridge at the time. Howe was a carpenter and the bridge was designed by average judgment, and the construction of one bridge made the precedent for another. Engineering science was not then applied to the calculating of stresses. Undue confidence in the form of truss, and the successful construction of many bridges in wood led to the Ashtabula accident. The strong desire for the construction of iron bridges at minimum cost, and the high price of iron, led to the construction of weak designs of this material, and postponed its adoption for many years. With the reduction of the price of iron, and a better understanding of the principles of construction, iron bridges have almost entirely superseded the wooden ones.

The forms have been calculated and tested by years of practical use,

and they are furnished by engineering bridge companies in lengths to suit, and guaranteed to resist the stresses required. The finding of the jury in the Bussey Bridge disaster on the Boston and Providence Railroad, is acknowledged to have been just, but this is not likely to occur with bridges as at present furnished, and subjected to careful inspection on the part of railroads. The limit of span has not yet been reached, nor perhaps the admissible height of skeleton piers, nor any form of truss or suspension established as the best. Cantilevers, from the ease of their erection without false-works, are largely used, but æsthetically in form they do not compare with the arch; yet extended use and knowledge of their "fitness for function" will soon reconcile one to the form.

Gas Engineering.

Much has been done by improvements in mechanics and chemistry to increase the production of gas and reduce its cost. Water-gas is largely manufactured, and its uses extended in forging and welding. It is to be hoped that it may be distributed by mains throughout cities for the purposes of heating buildings and cooking. And although from experiment it has been demonstrated that water-gas has but 65 per cent. of the thermic value of the coal from which it is manufactured, yet when the cartage and the storage of coal is taken into account, together with the curtailing of the hours of consumption by gas to the time necessary for the purpose, and the economy in the application of gas, there seems to be a fair presumption that, sooner or later, water-gas will be used for heating and cooking. In fact, from the great waste of heat in lighting by gas, and its cost, there perhaps might be some economy in converting the heat of water-gas into power through an engine, and into light by a dynamo, rather than directly by carburetted hydrogen into light.

Sanitary Engineering.

Of late years this branch of the profession has been recognized. It embraces the construction of sewers, the removal of sewage and its purification, or rather offense from it, and the plumbing and heating and ventilation of houses. The construction of sewers has long belonged to the department of hydraulic engineering, sometimes by the combined and sometimes by the separate system, the general aim being to remove sewage speedily from the town or city, and to points remote from settled

neighborhoods, and, if possible, discharge it into a large body of flowing water. Of the offense in the matter of smell from sewage there is no doubt; but that it is so dangerous to life and detrimental to health as is often claimed, I have just doubt from a large experience and from evidence of experts before court.

I have said in the earlier part of my address that the engineer is born, like the poet, sculptor, painter or any other artist, but that he is cradled on palpable material and nursed on facts, not metaphysical ideas, of "I think, therefore, I am," or even the syllogism of the second order of difference, "I think I think, therefore I think I am;" but "I am an engineer, and must think and work." It is not necessary for him that he should believe either in the corpuscular or the wave theory of light, in the atom whose weight is 1, the first digit in the 22d place of decimals of a gram, that pressure is due to the immense velocity of atoms in motion, or that even bodies fall by attraction of gravitation or by some later discovered force. If these present theories enable him to recollect facts, they serve their purpose. It is sufficient for the engineer to know the laws of force to be able to resist or use them; $v = gt$ or $v^2 = 2gs$ supplies him with the laws of gravitation even if he cannot understand the why.

The works of the engineer involve higher responsibilities than any other profession; he is to build, suitably, with materials at his disposal, solidly and securely for the prevention of accidents and for the protection of life, and economically in cost. He is to draw from his own and the experience of others, but he should not try experiments at another's cost without a full understanding with the parties employing him.

As an expert he must testify to facts, and the party who retains him must accept his facts, as they may be brought out in evidence, whether they are for or against them. It were better that the expert were attached to the court and not retained by either plaintiff or defendant. The construction of the law depends on the ruling of the judge. The value of the facts, from his point of view, should be settled by the engineer. As an umpire between the capitalist and contractor his decision should be impartial, and if, by his own error, either party has been put to an unnecessary expense, acknowledge the fault, and, as far as possible, remedy it, but where such responsibility is taken, the price should be commensurate.

From the position of the engineer as umpire between the capitalist

and contractor, and his large employment and familiar intercourse with mechanics and laborers, he would naturally be the proper party to adjudicate in disputes between labor and employer, and perhaps it would be so were the present troubles between labor and capital. But the strife is to see whether labor cannot support another class: chapters, presidents, officers, walking delegates and the like. These troubles come up in every country and in every age; they are like the measles, whooping cough, fever, cholera, in the microcosm. Ours, at present, is an imported disease; the diagnosis of the suppression of the individual, of boycott, etc., are entirely un-American. It is not our own labor, but the imported labor that is striving, not for a chance for labor, but rather for a life without labor. There is no country in the world where labor is so well paid, well clothed, well fed, and well housed. We have taken in an immense immigration, and still are, and have endeavored, by making them citizens, to assimilate them rapidly into our body corporate. This should be a question of time. It is a common phrase to say one has bitten off more than he can chew. How much more sad our case when we have swallowed more than we can digest! The great panacea seems to be land; some of us have worked land, and think if land were given to us instead of bread it would be the gift of a stone, but if properly applied it might relieve our present labor trouble. Let every immigrant be sent at once into the country and furnished a cabin and ten acres of land, without tax, as there is to be hereafter no title to land, and support himself and family ten years on the land. At the end of that time the majority would prefer the work and wages of a day laborer, especially in the city, and all would be better citizens. Syndicates of labor are as proper organizations as syndicates of capital or of employers, and syndicates can only be met by syndicates. Properly organized, they are not expensive and are efficient; but individuals should have perfect freedom to do as they please with their labor or capital within the limits of law, and must be protected by law and a healthy public sentiment.

The Secretary has furnished the following statement of the present (June 22) membership of the Society:

	Resident.	Non-Resident.	Total.
Honorary Members	3	5	8
Corresponding Members	...	3	3
Members	161	632	793 ...
Associates	11	34	45 ...
Juniors	28	92	120 958
	203	766	
Total			969
Fellows, 64, of whom 6 members are included above, leaving			58
Subscribers to the Building Fund, 140, of whom 90 are entered in one or other of the above classes, and 11 deceased, leaving			39
Total now connected with the Society			1066

Whence are we to draw our future membership to maintain the ranks of our Society? In answer, a distinguished professor and engineer has kindly furnished me the following statistics:

"There are in the United States 360 colleges and universities, which, besides classical and other courses, teach technology, and have given diplomas on technical professional subjects to 1 058 graduates. There are besides 49 schools, working under the 'Land Grant,' with 593 teachers and 4 644 students, of whom about 600 were technical graduates. In addition to these there are 39 private technical institutions with nearly 300 teachers and over 7 000 students, 800 of whom received technical diplomas. Statistics of this kind are very difficult to obtain, but it may be said that our higher schools of science turn out about 2 500 men per year, and of these about seven per cent. are engineers."

It becomes us, who are conversant with the needs of our profession, to advise that before entering our technological institutes, students should have a good school training, such as is required in entering classical universities, and that within the institutes they shall know not only what to learn, but also how to learn. Then in their outer world progress they may know how to accumulate knowledge useful to themselves in their profession, and reciprocate for the information and pleasure of those with whom they are thrown in contact.

In the earlier part of my address I claimed for the origin of our name, Engineer, *le genie*—genius; but genius, inborn, is derived from the Greek *gune*, woman, and I trust that there is no profession which has a higher respect or greater love for that name than our profession, whether as mother, sister, wife, or companion in the halcyon days of youth, or, in maturer years, our origin, protection, comfort, and inspiration. We trust that they will always attend our annual outings, intended not only for interchange of views in matters of our profession, but also as a relief from business, a union for the establishment of a more general acquaintance, and cementing the bonds between the members of the American Society of Civil Engineers.

AMERICAN SOCIETY OF CIVIL ENGINEERS.
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TRANSACTIONS.

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ON THE CALCULATION OF THE STRESSES IN BRIDGES FOR THE ACTUAL CONCENTRATED LOADS.

By GEORGE FILLMORE SWAIN, Assoc. Am. Soc. C. E.

READ MAY 4TH, 1887.

The actual loads to which railroad bridges, and, in a lesser degree, street bridges, are exposed, consist of a series of concentrated loads. Therefore the most accurate method of calculating the stresses in such structures would be to take the assumed or prescribed system of actual loads, find its position on the structure necessary to cause the maximum stress in any particular member, and determine in this way the maximum stress in each piece. Up to within about ten years this method was rarely followed, the usual method (except for the floor system) having been to assume a uniform load over the entire length covered by the engines and train; or, better, such a uniform load with the addition of concentrated loads over the drivers or at the head of the train, constituting the so-called locomotive excess. And it seems to the writer that this method is as accurate as any, while much more simple than that which depends upon the actual loads; for, in the first place, the weights of loaded cars have of late years so increased as to at present almost equal the heaviest engine loads, while the weights and dimensions of engines themselves are continually changing, so that, by assuming the load to consist of the concentrated loads of two particular engines followed

by a uniform train load, we are in reality, so far as the actual service of the bridge in question is concerned, assuming a system of loading almost as fictitious as a uniform load with locomotive excess would be. Nevertheless, many of the latest and best specifications require the use of the actual loads for engines and tenders, followed by a uniform train load, and this method seems destined to be for some time a favorite one. It therefore becomes necessary to investigate the stresses due to systems of actual loads, and the object of this paper is to give a discussion of the simplest methods of finding such stresses, with the corresponding positions of the system of loads. For the sake of completeness some methods will be given which are well known to bridge engineers, while others will be explained which it is believed will be found new to the profession in this country, and some which are original with the writer. It seems to be the belief of many that it is very much more difficult to calculate with the actual loads than with the locomotive excess; a belief which the writer hopes he may show to be erroneous in a great many cases. Considerable misconception, moreover, appears to still exist with regard to the exact position of the loads causing maximum stresses in different members, notwithstanding the fact that it requires in most cases but a very few moments to determine exactly the proper position required, even in the case of some structures which may in general be called complicated, like arches and cantilevers.

The writer is not acquainted with any systematic and complete treatment of this subject in English, and he therefore offers this paper with the hope of in some measure supplying this deficiency.

Commencing then with simple cases, let us proceed to consider moments and shears.

I. Maximum Moment at Any Point on a Beam.

A. *Without Floor-Beams.*

Considering the beam $A\,B$ in Fig. 1, Plate I, suppose we wish to find the maximum moment at any point c, situated at a distance x from the left abutment. Let the loads on the bridge be as represented, G_1 being the resultant of all the loads to the right of c, and G_2 that of all the loads to the left, G_1 acting at a distance n_1 from the right abutment, and G_2 at a distance n_2 from the left abutment. Then the moment at c is

$$M = G_1 \frac{n_1\, x}{l} + G_2 \frac{n_2\,(l - x)}{l} \qquad (1)$$

If now the entire system of loads be moved a distance $\triangle$ to the left, the moment at C becomes

$$M' = G_1 \frac{x(n_1 + \triangle)}{l} + G_2 \frac{(n_2 - \triangle)(l - x)}{l} \qquad (2)$$

and M' is greater than M if

$$G_1 x > G_2 (l - x)$$

or if

$$\frac{G_1}{l - x} > \frac{G_2}{x} \qquad (3)$$

But $\frac{G_1}{l-x}$ is the average load per running foot to the right of c, and $\frac{G_2}{x}$ is the average load per running foot to the left of c. Hence the rule:

If the average load per running foot to the right of the section is greater than the average load per running foot to the left of the section, then the moment at the section will be increased by moving the system of loads to the left, and *vice versa*. In other words, the maximum moment at any given point will occur when the average loads to the right and left are as nearly equal as possible. But with concentrated loads we shall be able only in exceptional cases to make the average loads on either side absolutely equal. We shall, however, always be able to place the loads so upon the bridge that with some particular load an infinitely small distance to the right of the section in question the average load to the right shall be greater than that to the left (thus showing that the loads should be moved to the left), while for the same load an infinitely small distance to the left of the section the average load to the left shall be the larger (thus showing that the loads should be moved to the right). From this it will follow that the load in question should be at the section; and from which it also follows that for the maximum moment at any section, some load must be at that section.

Suppose that with some load P_n just to the right of the section, we find the average load to the right larger than that to the left; we then move P_n just to the left of the section, and if now the average load to the right is still greater than that to the left, we then move the next load P_{n+1} up until it is just to the left of the section, and repeat the trial, regardless whether, during the operation, new loads come upon the bridge to the right, or leave it at the left. For, when P_n is just to the left of the section we move to the left, because the average load

to the right is greater than that to the left; and if, in moving the loads to the left, additional loads come upon the bridge at the right, then the average load to the right is so much larger still, and, *a fortiori* we must continue to move to the left. The same form of argument applies when loads leave the bridge to the left.

This method will not give, of course, with absolute certainty the position of loading causing maximum moments; for it sometimes happens that two or more different loadings will be found which will satisfy the condition just explained, and to find which of these gives the absolute maximum moment, each must be worked out numerically. By remembering, however, that to produce the maximum moment at any point, the heaviest loads must be as near that point as possible, the cases in which trial is necessary will be greatly reduced in number.

B. *With Floor-Beams.*

In the case of truss bridges it is only necessary to find the moments at the joints, but at some of these there may be floor-beams, and at others there may be none. It is, however, rarely necessary to consider the floor-beams at all, for the reason that, as is well known, the moment at the floor-beams is precisely the same whether floor-beams be supposed to exist or not, while at points between the floor-beams the actual moment (when floor-beams exist) is slightly less than the moment which would exist were there no floor-beams. It is therefore in almost all cases sufficient to calculate as though there were no floor-beams at all, but as if the loads were applied directly to the girders. Methods will be soon explained, however, by which the effect of floor-beams at intermediate points may be considered if desired.

II. Maximum Shear at any Point on a Beam.

A. *No Floor-Beams.*

Suppose that we desire to find the maximum positive shear at any point c of a beam (Fig. 2, Plate I). It is well known that for the maximum positive shear the portion $c\,B$ of the beam must be loaded, and that as a train comes on the bridge from the right, the shear at c will increase at any rate until the first wheel reaches c. If the first wheel passes c the shear suddenly diminishes by the amount of the load P_1, but if the train move still farther to the left it will increase again until P_2 reaches c. The question now is: Will the shear be greater when P_2 is at c than

when P_1 is at c? This question is easily answered: Let R be the resultant or sum of all the loads on the bridge when P_1 is at c, then the shear at c is the left-hand reaction R_1 or $R\frac{n}{l}$. If now the train be moved a distance a_1 to the left, so that P_2 is just to the right of c, the shear will be $R\frac{n+a_1}{l} - P_1$ plus some additional small quantity d, which is the left-hand reaction due to any additional loads which may have come on the bridge during this advance of the train. The shear will therefore be increased by moving up P_2, provided

$$R\frac{a_1}{l} + d > P_1 \tag{4}$$

or (since we can almost always neglect d), provided

$$R\frac{a_1}{l} > P_1 \tag{4A}$$

It is only necessary to consider d when these two terms are very nearly equal, which does not very often occur.

Similarly, the shear would be increased still further by moving P_3 up to c, provided

$$R_1\frac{a_2}{l} > P_2 \tag{5}$$

R_1 being the sum of the loads on the span when P_2 is at c.

B. *With Floor-Beams.*

Let the distance between floor-beams (Fig. 3) be called p. Then since the shear at any section c is equal to R_1 minus all the loads to the left of c, and since no loads act on the main girder except through the floor-beams, it follows that the shear between any two adjacent floor-beams is constant. The question therefore is to find the maximum positive shear in any panel, such as $C\,D$.

As a train approaches from the right, the shear will be simply the reaction R_1 as long as all the loads are to the right of D, and it will continually increase until the first load lies just at D. If now the train is moved farther, the shear will be equal to R_1 minus the load which comes on the floor-beam C_1, and will either increase or decrease uniformly until P_2 arrives at D, or until P_1 passes C. The question is therefore as before: Will the shear be greater with P_1 or P_2 at D? This is easily determined, for when P_1 is at D the shear is $R\frac{n}{l}$; while with P_2 at D it is

$$R\frac{n+a_1}{l}+d-P_1 \text{ (if } a_1 > p)$$

or,

$$R\frac{n+a_1}{l}+d-P_1\frac{a_1}{p} \text{ (if } a_1 < p)$$

Hence the shear is greater when P_2 is at D, provided

$$R\frac{a_1}{l}+d > P_1 \qquad \text{(if } a_1 > p) \tag{6}$$

$$R\frac{a_1}{l}+d > P_1\frac{a_1}{p} \text{ (if } a_1 < p) \tag{7}$$

or, neglecting d as before, provided

$$R\frac{a_1}{l} > P_1 \qquad (a_1 > p) \tag{6A}$$

$$R\frac{a_1}{l} > P_1\frac{a_1}{p} \ (a_1 < p) \tag{7A}$$

In these equations, which are extremely easy of application, R represents, as before, the sum of all the loads on the bridge when P_1 is at D.

Having found, perhaps, that the shear is greater with P_2 at D than with P_1 at D, the question may arise: Will it be greater still with P_3 at D? Calling R' the sum of the loads on the bridge when P_2 is at D, the condition in this case will be easily seen to be the following: The shear will be greater with P_3 at D than with P_2 at D, provided

$$R'\frac{a_2}{l}+d > (P_1+P_2)\frac{a_2}{p} \qquad \text{(if } (a_1+a_2) < p) \tag{8}$$

$$R'\frac{a_2}{l}+d > P_1\frac{p-a_1}{p}+P_2\frac{a_2}{p} \qquad \text{(if } a_1 < p,\ a_2 < p, \text{ and } a_1+a_2 > p) \tag{9}$$

$$R'\frac{a_2}{l}+d > P_1\frac{p-a_1}{p}+P_2 \qquad \text{(if } a_1 < p,\ a_2 \tag{10}$$

$$R'\frac{a_2}{l}+d > P_2\frac{a_2}{p} \qquad \text{(if } a_1 > p,\ a_2 < p) \tag{11}$$

$$R'\frac{a_2}{l}+d > P_2 \qquad \text{(if } a_1 > p,\ a_2 > p) \tag{12}$$

In most cases, as before, we may neglect d.

III. Lines of Influence.

We proceed to explain a method by which shears and moments, as well as stresses, deflections, etc., may be calculated, and which, by its perfect generality, may be applied to the solution of a great variety of problems which could scarcely be solved in any other way.

To show what a line of influence is, let us consider first the case of moments on a beam, and we shall see how the results already reached follow very simply from the properties of the influence line.

Let $A\ B$ (Fig. 4, Plate I) represent a beam supported at A and B, and suppose we wish to find the maximum movement at the point c. To do this, imagine a single load equal to unity to pass over the bridge from right to left, and at each position of the load let us lay off as an ordinate the moment which would exist at c. Thus, $m\ n$ represents the moment at c which would be caused by a load of unity at m, and $c\ d$ that which would be caused by the load unity at c. The $A\ d\ B$, which is the influence line for the moment at c, evidently gives the moment for any and every position of the load; $c\ d$ is evidently equal to $\frac{x\,(l-x)}{l}$, and $A\ d'$ and $d\ B$ are straight lines, which may be drawn by first laying off $c\ d$ to some scale.

Now suppose the actual loads on the bridge are as represented in Fig. 5, and that we desire to find the moment at c, clearly this moment will be

$$M = P_1\ y_1 + P\ \ y_2 + P_3\ y_3 + P_4\ y_4 + P_5\ y_5 + P_6\ y_6 + P_7\ y_7 \quad (13)$$

From the influence line we can therefore obtain this moment at c for any position of actual loads on the span.

Suppose, again, that the load consists of a uniform load of p pounds per running foot. The influence line may be used as before; for in Fig. 6, taking an infinitely small distance $d\ x$ at g, the load on this distance will be $p\ d\ x$, and the moment it will cause at c will be $p\ y\ d\ x$, since a load of unity at g would produce a moment y at c.

Here the moment caused by the uniform load over any portion $m\ m'$ of the span will be $\int_{x_1}^{x_2} p\ y\ d\ x = p \int_{x_1}^{x_2} y\ d\ x = p\ A$, A representing the area $m\ n\ n'\ m'$. The moment caused by the load over the entire span will be $p\ A$, A being the entire area $A\ d\ B$, or it will be $M = \frac{1}{2}\ x\ (l-x)$, since the ordinate $c\ d$ is, as already explained, $\frac{x\,(l-x)}{l}$

The influence line enables us therefore to find the moment due to any loading whatsoever. It also shows us how much of the beam must be loaded in order to produce the maximum moment, where a single load must be to produce the maximum moment, and it may be used in a variety of problems, as will soon appear.

Suppose now we wish to find how a system of single loads must be placed on a beam to cause the maximum moment at c. We have already investigated this problem, but we shall now show how the same results follow from the properties of the influence line, and how we are thus enabled to draw some further conclusions. In Fig. 7, Plate I, let $A\,d\,B$ be the influence line as before, and let G_1 and G_2 be the resultant loads to the right and left of c respectively. Now if the entire system of loads be moved a distance $d\,x$ to the left, then any load P lying at m, and which at first caused a moment represented by $P\,y$, will now cause a greater moment, namely $P\,(y + d\,y)$, or its moment will be increased by $P_1\,d\,y = P\,d\,x$ tan. α_1. The same will be true of all the loads G_1, while all the loads G_2 will have the moments decreased by $P\,d\,x$ tan. α_2. The resultant moment at c will therefore be increased by moving the system to the left, provided

$$G_1\,d\,x \text{ tan. } \alpha_1 > G_2\,d\,x \text{ tan. } \alpha_1, \text{ that is if}$$

$$G_1 \text{ tan. } \alpha_1 > G_2 \text{ tan. } \alpha_2, \text{ or if}$$

$$\frac{G_1}{l - x} > \frac{G_2}{x}$$

as we showed on a previous page.

Influence lines may be drawn not only for the moment at a point, but for the shear, or for the stress in any bar of a frame; or for the deflection, or any other function of the loads which we desire to investigate. From the theorem just proved, one very fertile principle follows, which finds numerous applications, namely, this:

Whenever the influence line for any function is composed of two straight lines, like those in Fig. 7, Plate I, then the maximum value of the function in question, for a system of concentrated loads, will occur when the load covers the length A B when some load is at c, and when the average load on A c is as nearly as possible equal to that of B c.

In other words, the loading causing the maximum value of the function will be the same loading which would cause the maximum moment at c on a beam of length A B.

Before proceeding to the applications of this principle, let us consider some other lines of influence.

Moment; with Floor-Beams.—As before remarked, we most frequently have to find the moment at the points where the floor-beams are supported, and at such points the moment being precisely the same,

whether there are floor-beams or not, the influence lines are the same. At points between the floor-beams, such as c in Fig. 8, Plate I, the influence line, as will easily be seen, is $A\ H\ G\ B$, the moment $c\ d^1$ being somewhat less than the moment $c\ d$, which would exist were there no floor-beams. The simplest way of drawing this influence line is to lay off $c\ d$, $A\ d$, and $d\ B$ as though there were no floor-beams, and then join G and H.

Lines of Influence for Shear.—Suppose we wish to study the shear at a point c on the beam $A\ B$ in Fig. 9. The influence line will evidently be $B\ d\ e\ A$, $c\ d$ being equal to $\frac{l-x}{l}$, and $c\ e = -\frac{x}{l}$. The straight lines $B\ d$ and $A\ e$ are parallel, and $d\ e = 1$. This line tells us at once that with a uniform load the maximum positive shear occurs when only $c\ B$ is loaded, and the maximum negative shear when only $A\ c$ is loaded; and, further, that

$$\text{Maximum } (+S) = \frac{1}{2}\, p\, \frac{(l-x)^2}{l}$$

$$\text{Maximum } (-S) = \frac{1}{2}\, p\, \frac{x^2}{l}$$

as is well known. With a system of concentrated loads, the shear will be given by equation 13.

If we wish to ascertain whether the positive shear will be greater with P_2 at c than with P_1 at c, we observe that if P_2 is moved up to c, P_1 will be moved forward a distance a_1 to the point g. If now the influence line were $B\ a'$, the shear would be increased by Ra_1 tan. α, but since P_1 passes c we must subtract $P_1\ (s\ t)$; hence the shear is really increased by Ra_1 tan. $\alpha - P_1\ (s\ t)$. But $s\ t = A\ a' = 1$; hence the shear is increased provided $R\, a_1$ tan. $\alpha > P_1\ (s\ t)$, or provided $R\ \frac{a_1}{l} > P_1$, as found on page 26. And here, as in the case of moments, we may conclude that whenever the influence line for any function is composed of two parallel straight lines connected by a vertical, the maximum value of the function will occur for the same loading which on a span $A\ B$ without floor-beams would cause the maximum shear at c.

Shear; with Floor-Beams.—A brief consideration will show that in this case the influence line for the shear in the panel $g\ h$ (Fig. 10, Plate I) is $B\ H\ G\ A$. When the load lies at k, the shear in the panel $g\ h$ is zero; when it lies to the right of k the shear is positive, and when it lies to the

left of k it is negative. The point k is such a point that when a load lies there the reaction at A will be equal to the load on the floor-beam at g, that is

$$P\frac{s}{l} = P\frac{z}{p} \text{ or } \frac{s}{l} = \frac{z}{p}.$$

This point k may be called the neutral point of the panel $g\,h$, and each panel has such a neutral point. The influence line gives us the point k directly, and shows that with a uniform load the maximum positive shear will occur when the load covers $B\,k$ alone, and the maximum negative shear when the load covers $A\,k$ alone. Calling A the area $B\,H\,k$, and A_2 the area $A\,G\,k$, we have

$$\text{Maximum } (+S) = p\,A.$$
$$\text{Maximum } (-S) = -p\,A.$$

Since $H\,h = \frac{x^1}{l}$, and $G\,g = -\frac{x^2}{l}$, the lines $A\,G$ and $B\,H$ are readily seen to be parallel.

If the loading consists of a series of concentrated loads, equation 13 is to be used.

Finally, we may again deduce from the influence line the conditions determining the loading giving maximum shear for a series of concentrated loads. Starting with the first wheel at h, we must determine whether the shear is increased by bringing up P_2 to h. If in moving the loads in this way P_1 passes beyond G; or, in other words, if $a_1 > p$, we have the increase of shear equal to Ra_1 tan. $\alpha_1 - P_1\ (s\,t)$, or, since $s\,t = A\,a_1 = 1$, the shear will be increased if $R\frac{a_1}{l} > P_1$. Similarly, if $a_1 < p$, and P_1 moves to $s'\,t'$, the increase of shear will be Ra_1 tan. $\alpha_1 - P\ (s'\,t')$; or the shear will be increased if $R\frac{a_1}{l} > P\frac{a_1}{p}$ as already shown on page 26. We may again conclude, therefore, that whenever the influence line is composed of two parallel straight lines joined by an inclined line, as in Fig. 10, the loading causing the maximum value of the function will be the same loading that on a beam of length $A\,B$ would cause the maximum shear in the panel $g\,h$.

As a last illustration of influence lines, consider that for the stress in the bar $a\,b$ of the truss shown in Fig. 11, Plate I. Since the stress in $a\,b$ is simply the moment at g divided by the height h, the influence line $A\,d\,B$ will be exactly similar to that for the moment at g, the ordinate $c\,d$ being $\frac{x\,(l-x)}{l\,h}$.

IV. Example of Maximum Shears and Moments.

Although the calculation of maximum shears and moments is generally well understood by bridge engineers, the following example will illustrate the simplicity of the following methods, and will add to the completeness of this article.

In finding moments and shears, a diagram like that in Fig. 12, Plate II, should be used, as is done in several bridge offices. Fig. 12 is made out for a Pennsylvania Railroad consolidation engine, the lower line giving the loads on each pair of wheels; the next line above the distances of each load from the first load of six tons; the third line the sum of all the loads up to and including the load above which the figure occurs; and the upper line the sums of moments, about any load, of all the loads on its left. Thus, $6 \times 7.5 = 45$; $6 \times 12 + 12 \times 4.5 = 126$ (or more rapidly, $45 + 18 \times 4.5 = 126$). With the aid of this diagram, moments may be very rapidly calculated.

Example.—Let it be required to find the maximum stresses in the bars of the truss shown in Fig. 13, Plate I, for the loads in Fig. 12.

Chord Bars.—For the maximum chord stresses it is only necessary to find the maximum moments at the points a, b, c and d.

Maximum Moment at a.—If we try the first driver just to the right of a, then if the locomotives are preceded, as well as followed, by a uniform load, as shown in Fig. 12, a portion of the panel $A\,a$ will be covered by this uniform load. We shall, however, not consider any load in front of the engines, as it is generally unnecessary in practice to do so, and ordinary specifications do not call for it. We then have the right-hand abutment B 105 feet from the load 2, or 112.5 feet from 1, and therefore 10.5 feet beyond 18, the last 7.5 feet being therefore covered by the uniform load. To the left of a we have six tons, to the right $166 + 7.5 \times 1.5 = 177.25$, and as there are seven panels to the right and one on the left, the average loads may be represented by $\frac{6}{1}$ on the left, and $\frac{177.25}{7} = 25 +$ on the right. As the average load on the right is greater, we must move the loads to the left. With the load 2 just to the left of a we shall have the average loads,

$$\text{On the left, } \frac{18}{1} = 18; \text{ on the right, } \frac{165.25}{7} = 23 +;$$

hence we must still move to the left. With the load 3 just to the left of

a, we have B 15 feet beyond the load 18, and 12 feet covered by the uniform load. The average loads are

$$\text{On the left, } \frac{30}{1} = 30; \text{ on the right, } \frac{160}{7} = 23+.$$

Hence the system must be moved to the right, and therefore the load 3 must lie at a.

This case has been worked through in detail, but with practice the portion of the loads causing maximum moments may be so nearly assumed in the beginning that but one or two trials are necessary to determine it exactly. It must only be borne in mind that the entire span must be loaded as much as possible, with one heavy load at the point in question, and with the heaviest loads as near that point as possible. In the case just treated we should have tried the load 3 at first, if we had not desired to illustrate the method of procedure.

It must be remembered, however, that several distinct portions of the load may give maximum moments at the same point, and to decide between them the only exact way is to work out the actual moments. They will, however, in many cases be very nearly alike, and inspection will almost always show which will probably be the greater. Thus, in the above case, if we try the load numbered 8 on the right and left of a, we shall find that when at a it gives a maximum moment; but in this case it is very evident that this second maximum will be smaller than the one corresponding to load 3 at a. In some cases, however, a trial may be necessary.

The maximum moment itself is easily found, the load 3 being at the point a. The reaction at A is found by taking moments about B, which lies fifteen feet beyond the load 18. This moment is $8\,934 + 172 \times 15 + 18 \times 6 = 11\,622$, the last term ($18 \times 6$) being the moment of the uniform load covering the last 12 feet of the span. The reaction is $\frac{11\,622}{120} = 96.85$ tons, and the moment at $a = 96.85 \times 15 - 126 = 1\,452.75$ foot tons. Finally, the stress in the bars $A\,a$ and $a\,b$ is $\frac{1\,452.75}{20} = 72.6$ tons for both trusses.

In using the schedule Fig. 12, care must be taken to allow for any of the front loads passing off the bridge, as will appear in what follows. The remaining moments may thus be calculated.

Moment at b.—In this case it will be found that the maximum moment occurs when load 5 is at b; but in all probability, since for this loading

the other drivers are all in front of b, a still greater moment will occur when the second engine reaches b. The trial shows, in fact, that a maximum occurs when load number 12 is at b. For this loading the span reaches from 37 to 157 on the line of distances, and we have

Reaction at left =

$$\frac{8\,934 + 172 \times 55 + 1\tfrac{1}{2} \times 52 \times 26 - 1\,327 - 70 \times 120.5}{120} = 88.83 \text{ tons.}$$

Moment at $b = 88.83 \times 30 - 126 - 8\,(20 + 25) = 2\,179$ foot tons.

$$\text{Stress} = \frac{2\,179}{20} = 108.95 \text{ tons (in } a'\,b' \text{ and } b\,c\text{).}$$

For the loading with load 5 at b we have

$$\text{Reaction} = \frac{8\,934 + 172 \times 9 + 1\tfrac{1}{2} \times 6 \times 3}{120} = 87.575$$

Moment at $b = 87.575 \times 30 - 450 = 2\,177$, or almost precisely the same as the other, though a little smaller.

Moment at c.—A trial shows that load 13 gives maximum moment. The span reaches from 26.5 to 146.5 on the line of distances.

Reaction =

$$\frac{8\,934 + 172 \times 44.5 + 1\tfrac{1}{2} \times 41.5 \times 20.75 - 450 - 54 \times 125.5}{120} = 88.77$$

$$M = 88.77 \times 45 - 261 - 32 \times 32.25 = 2\,701.65.$$

$$\text{Stress in } b'\,c' \text{ and } c\,d = \frac{2\,701.65}{20} = 135.08 \text{ tons.}$$

Moment at d.—The load 12 lies at the point, the span reaching from 7 to 127, and with 22 feet loaded with the uniform load.

$$R = \frac{8\,934 + 172 \times 25 + 1\tfrac{1}{2} \times 22 \times 11 - 6 \times 127}{120} = 106.96$$

$$M = 106.96 \times 60 - 3\,948 - 6 \times 67 = 3\,321.6$$

$$\text{Stress} = \frac{3\,321.6}{20} = 166.08 \text{ (in } c'\,d'\text{).}$$

SHEARS.—*Panel A a.*—With P_1 at a, the total load on the span is 172 tons. Now since $172 \times \frac{7.5}{120}$ is greater than $6\,\frac{7.5}{15}$, the shear will be increased by bringing up P_2. If P_2 be brought up, P_1 will still remain on the bridge; the reaction will be increased by $183.25\,\frac{4.5}{120} + d$ (on account of the uniform load coming on, d being exactly equal to $1\tfrac{1}{2} \times 4.5 \times \frac{2.25}{120}$), or neglecting d, by 6.87. But we must substract $\frac{4.5}{15}.18$ more than when P_2 is at a; hence since 6.87 is greater than this (or 5.4), the

shear will be still farther increased if P_3 is brought up to a. It could in the same way easily be found that if P_4 were advanced to a the shear would be diminished, regard being had to the fact that here P_1 would pass off the bridge. The maximum shear, with P_3 at a, is found as follows, the span ending at 117 feet from P_1, and the last twelve feet being covered by the uniform load:

$$\text{Reaction at left} = \frac{9\,450 + 172 \times 12 + 18 \times 6}{120} = 96.85$$

$$\text{Shear} = 96.85 - \frac{12}{15}.6 - \frac{4.5}{15}.12 = 88.45 \text{ tons.}$$

Panel a b.—With P_1 at b, $R = 148$; and since $148 \frac{7.5}{120} > 6 \frac{7.5}{15}$ [or $148 > 6 \frac{120}{15} (= 48)$] the shear is increased by bringing up P_2. Again (the total load being now 164), since $164 \frac{4.5}{120} (= 6.15) > \frac{4.5}{15}.18$ $(= 5.4)$, it is still further increased by bringing up P_3. Finally, since $172 \frac{4.5}{120} (= 6.45) < \frac{3}{15}.6 \times \frac{4.5}{15}.24 (= 8.4)$, it is diminished by bringing up P_4. Hence P_3 must lie at b, and

$$\text{Maximum shear} = \frac{8\,934}{120} - \frac{12}{15}.6 - \frac{4.5}{15}.12 = 66.05$$

Panel b c.—With P_1 at c, the total load $R = 128$, and since $128 > 48$, we must bring up P_2. And since $140 \frac{4.5}{120} (= 5.25) < 5.4$ (as above), we must not bring up P_3. Hence P_2 must be at c, and

$$\text{Maximum shear} = \frac{5\,046 + 140 \times 6.5}{120} - \frac{7.5}{15}.6 - 45.88.$$

Panel c d.—With P_1 at d, $R = 92$, and since $92 > 48$, we must bring up P_2. It is not necessary to try P_3, since in the previous panel it was not advanced to c. With P_2 at d,

$$\text{Maximum shear} = \frac{3\,948 + 116 \times 0.5}{120} - \frac{7.5}{12}.6 = 29.63$$

Panel d e.—With P_1 at e, $R = 78$; and since $78 > 48$, we must advance P_2, and

$$\text{Maximum shear} = \frac{2\,102 + 86 \times 5.5}{120} - \frac{7.5}{15}.6 = 18.46$$

Panel e f.—With P_1 at f, $R = 54$, hence P_2 must be advanced.

$$\text{Maximum shear} = \frac{1\,327 + 70 \times 1}{120} - \frac{7.5}{15}.6 = 8.64$$

Panel f g.—With P_1 at g, $R = 30$, hence P_1 must be at g.

$$\text{Maximum shear} = \frac{126 + 30 \times 3}{120} = 1.8.$$

We have thus found the maximum positive shear in every panel.

VI. Single Intersection Trusses.

Little more than the processes used in the example just given is necessary in the calculation of ordinary single intersection trusses with parallel chords. The chord stresses are found from the moments, the web stresses from the shears. In finding chord stresses in Warren girders however, it is necessary, in order to obtain the stresses in the loaded chord, to calculate maximum moments at the opposite joints of the unloaded chord, that is, at points between the floor-beams. In this case however, as already remarked, it is sufficient to calculate exactly as though there were no floor-beams present, the results being then a little too large; but, if desired, influence lines may be used, with the application of equation 13. For example, in Fig. 11, Plate I, to find the maximum stress in the bar d, it is necessary to find the maximum moment at the point a. The influence line for the stress in d is given in Fig. 14; and calling G_1 the total load on $g\,B$, G_2 that on $h\,g$, and G_3 that on $A\,h$, then the stress in d will be increased by moving the loads to the left, so long as

$$G_1 \tan.\ \alpha_1 > G_2 \tan.\ \alpha_2 + G_3 \tan.\ \alpha_3$$

or, substituting for tan. α_1, tan α_2, tan. α_3, their values, so long as

$$G_1 > G_2 \frac{l-2\,t}{2\,t} + G_3 \frac{l-t}{t} \quad \text{or}$$

$$G_1 + G_2 + G_3 > G_2 \frac{l}{2\,t} + G_3 \frac{l}{t} \qquad (14)$$

The position of the load giving maximum stress being found, use equation (13).

Further, it is necessary to consider the effect of counters on the use of influence lines. Thus, in Fig. 13, suppose that there is no counter $c\,d'$, the influence line for the diagonal $c'\,d$ will be $A\,C\,D\,B$ in Fig. 15. If, however, the counter be in existence, the true line will be $A\,F\,D\,B$. In calculating diagonals, however, by means of influence lines, it is clear that we must treat each one as though there were no other diagonal in the same panel, or using for $c'\,d$ the line $A\,C\,D\,B$. For, if the diagonal is in action, every load on the span affects it, causing either tension or compression, and if the resultant stress for any particular loading is

compression, it simply shows that the diagonal considered would for that loading not be in action, but that the other one would be; and if this last is treated in the same way, the load in question will be found to produce tension in it. Similarly, in calculating chords and verticals, the influence line must be drawn on the supposition that certain diagonals are in action. Then, for any particular loading, it must first be determined whether such diagonals are really in action or not, and if not, then in getting the chord or vertical stress for such loading, the influence line used must be one constructed on the basis of the diagonal which is really in action.

Let us now proceed to some applications of these principles.

VI. Application to Floor-Beams, Floor-Beam Hangers, End Suspenders, etc.

In the truss shown in Fig. 13, the greatest load on any floor-beam, as a, occurs for some particular position of the loads, which should be determined, as upon it will depend the calculation of the floor-beam and its connections with the stringer, of the floor-beam hangers, and of the end suspender or hip vertical $a\,a'$. Fig. 16 represents the influence line for the load on the point a, the ordinate $n\,n'$ being unity. This line is exactly similar to that for the moment at a on a beam of length $A\,b$; hence the loading which would cause the maximum moment in the latter case will also cause the maximum load on a. Hence we see that to cause the greatest stress on floor-beams, hangers or hip verticals, the two adjacent panels should be loaded in the same way as would give the maximum moment at the floor-beam considered on a span equal to the two adjacent panels. This simple theorem applies to all cases of posts or ties such as $a\,b$ in Figs. 17, 18, 19 and 20, Plate I.

Let us consider further the strut $c\,d$ in Fig. 20. The influence line is shown below, $n\,n'$ being unity, and it is therefore easily seen that for the maximum stress the loading should be such as would give the maximum moment at c on a beam of length $A\,f$. The same is true regarding the tie $c\,d$ in Fig. 19.

For the strut $f\,g$ in Fig. 20, the influence line is shown below, $n\,n'$ being unity, and it follows that the loading giving maximum stress will be the same which would give the maximum moment at f on a span $A\,B$.

Moreover, the simplest way of finding the stress is to find actually, by means of the diagram Fig. 12, the maximum moment for the corre-

sponding loading, and then to multiply it by the ratio which the ordinate of the influence line for the actual stress bears to the ordinate of the influence line for moment. For example, in Fig. 16 the ordinate $n\,n'$ of the line for stress is 1, while for moments it would be $\frac{15}{30}.15 = 7.5$. Hence the maximum stress in $a\ a'$ in tons will be equal to the maximum moment at a on a beam $A\,b$, in foot tons divided by 7.5. Similarly, for $c\ d$ in Fig. 20, supposing the panels to be 20 feet long, the ordinate for stress is 1, and for moments $\frac{40}{80}\ .40 = 20$; hence

$$\text{Maximum stress in } c\,d \text{ (tons)} = \frac{1}{20}\,(\text{max. moment at } c \text{ in foot tons}).$$

VII. Fink and Bollman Trusses.

These trusses are taken up here as offering some analogy to cases just discussed. The loading causing maximum stresses in the verticals has already been fully explained, together with the method of determining such stresses. The stresses in diagonals follow at once from those in the verticals, and need not be further discussed. In considering the chords, an interesting application of influence lines may be made.

Fig. 21, Plate II, represents a Fink truss in which we have to find the maximum chord stresses. The influence line for the bar a is the one in full lines in Fig. 21A, and the ordinates may be easily checked. From this the loading causing maximum stress may be easily found by trial, using equation 13, but a rule may also be deduced which will save time.

Let G_1 = resultant load on ei.

G_2 = " " ce.

G_3 = " " bc.

G_4 = " " ab.

Starting with any given position of the loads, if the system be moved to the left a distance $d\,x$, it will be easily seen that the stress will be increased if

$$G_1 \tan.\ \alpha_1 > G_3 \tan.\ \alpha_2 + G_4 \tan.\ \alpha_3; \quad \text{or if}$$

$$G_1 > G_3 + 3\,G_4 \qquad (15)$$

By the aid of this condition, the loading causing maximum stress may be readily found, and then the actual stress by equation 13.

The influence line for the bars b is the one dotted in Fig. 21A. In this case, if we call G_1, G_2, G_3, the total loads on $d\,i$, $c\,d$ and $a\,c$ respect-

ively, the loading causing maximum stress will be found by moving to the left if

$$G_1 > G_2 + 2\,G_3 \tag{16}$$

In the Bollmann truss shown on Fig. 22, Plate II, the stress in the upper chord is constant throughout, and the influence line is given in Fig. 22A. The vertices all lie on a parabola. If the loads on each panel are called $G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8$, we may find the loading causing maximum stress by the condition that the loads must be moved to the left, provided

$$7G_1 + 5G_2 + 3G_3 + G_4 > G_5 + 3G_6 + 5G_7 + 7G_8 \tag{17}$$

VIII. The Truss with Intermediate Secondary Systems.

Under this head we include trusses like those shown on Figs. 23, 24 and 25.

In Fig. 23, Plate II, the stress in a will be obtained by finding the maximum moment at B and dividing by h; that in b by finding the maximum shear in the panel DB; that in c as already explained. The vertical component of that in d will be one-half that in c; for in Fig. 23c we have at the joint K the equations of equilibrium as follows:

$$S_1 \sin.\ \alpha + S_4 \sin.\ \alpha - S_2 \sin.\ \alpha = o \tag{18}$$

$$S \cos.\ \alpha - S_4 \cos.\ \alpha - S_2 \cos.\ \alpha - S_3 = o \tag{19}$$

from which we obtain

$$S_4 = \quad -\frac{1}{2} S_3 \sec.\ \alpha \tag{20}$$

$$S_1 = S_2 + \frac{1}{2} S_3 \sec.\ \alpha \tag{21}$$

To find the maximum stress in e, draw its influence line, as in Fig. 23A, which is drawn for the vertical component of the stress in e, regardless whether there be a counter, as in the panel to the right. It will be found that this influence line is exactly the same as it would be if there were no bars c and d. The ordinates at B' and C' are clearly $\frac{s}{l}$ and $\frac{s'}{l}$ respectively, and if G and H be joined by a straight line, the ordinate to this line at D' will be $\frac{s}{l} - \frac{1}{2}\frac{(s+s')}{l} = \frac{1}{2}\frac{s-s'}{l}$; but this can be shown to be the ordinate to the influence line at D' by considering that if a load of unity lies at D, the shear in the panel CD is $\frac{s + \frac{l-s-s'}{2}}{l}$ while

the shear in the bar d is, according to equation 20, a compression of $\frac{1}{2}$, hence the shear in the bar e is

$$\frac{s + \frac{l - s - s'}{2}}{l} - \frac{1}{2} = \frac{s - s'}{2\,l}$$

Hence the loading causing the maximum stress in e, as well as that maximum stress itself, is found entirely regardless of the secondary systems, just as though the truss were an ordinary Pratt truss with joints only at C, B, etc.

The stress in f is the same as that in g. To find it, draw the influence line, as in Fig. 23B. $A_1\,C\,B\,B_1$ is the line we should have if there were no secondary system, and the ordinates at C' and B' must clearly be the same in the present case. We have only to find the ordinate at D', since between any two consecutive joints the line must always be straight. With a load unity at D, we find the stress in f by taking moments about O, allowing for the moment of the stress in d. Thus,

$$\text{Reaction on the left} = \frac{s + \frac{1}{2}p}{l} \qquad (p = \text{panel length}).$$

$$\text{Hence component of stress in } d = \frac{1}{2}\frac{p}{h} \qquad \therefore$$

$$\frac{s + \frac{1}{2}p}{l} \cdot s' + \tfrac{1}{2}\frac{p}{h} \cdot h = g\,h \qquad \therefore$$

$$g = \frac{s\,s'}{l\,h} + \frac{s'\,p}{2\,l\,h} + \frac{p}{2\,h} \tag{22}$$

The ordinate to $C\,B$ at the point D' would be

$$y = \frac{s\,s'}{l\,h} + \frac{s'\,p}{2\,l\,h}$$

hence the point D is above the line $C\,B$ by the distance $\frac{p}{2\,h}$. In other words, the stress in g and f is equal to that which would exist if there were no secondary system, plus the stress in $C\,B$ considered as the chord of the secondary truss $C\,K\,B$. The ordinate at D' will be the longest in the curve, even greater than $C\,C'$ in every case. The loading causing maximum stress will be nearly the same as if there were no secondary system, but there will generally be a load at D.

The stress in the vertical, h, is readily found, being the vertical component of that in the diagonal e, plus whatever load may be on O. Since the stress in e is the same as without the secondary system, the stresses in the verticals will be also. The same is true of the counters.

Fig. 24, Plate II, represents a deck bridge of the same system as Fig. 23, and the same laws hold as for the bars bearing the same letters in Fig. 23.

Fig. 25, Plate III, represents a triangular truss with three intermediate verticals in each long panel $A\,E$. The stresses in this truss may be found as follows:

That in a equals the maximum moment at A divided by b, the loading being the same as would cause the maximum moment at A on a simple beam of the same span.

Those in b, b', and d are found as has been already explained.

That in c and c' is equal to one-half of that in b, or b', resolved along the bars.

That in e has for its vertical component the maximum shear in the small panel $A\,B$, the beam being treated as a simple beam with panels equal to $A\,B$.

That in e' is found in a similar way, from the shear in $D\,E$.

For f' the influence line is drawn in Fig. 25A, showing that the stress is just the same as in a truss with panels equal to $A\,C$ and $C\,E$, that is, disregarding the small systems $A\,G\,C$, $C\,K\,E$, etc. The same is true of f.

For g and h the stress will evidently be the same as in the bar $A\,C$ of a truss with panels equal to $A\,C$, $C\,E$, etc., plus the stress in $A\,C$ as the chord of the small truss $A\,G\,C$. The influence line may be easily drawn and the maximum loading will, as in Fig. 23, generally be with some load at B. Similar remarks apply to the bars k, l.

IX. Trusses with Multiple Systems of Web.

Trusses belonging to this class—double intersection trusses, as they are commonly built in this country—have always offered some difficulties in calculation for the actual loads. If a uniform load be assumed, with locomotive excess, the calculation is very simple, for the load coming on each of the systems of which the frame consists is definitely determined. When it comes, however, to consider the actual loads, it is by ordinary methods not easy to tell just what loading causes the maximum stresses; although, having that loading, the stresses themselves may be found without difficulty. The writer believes that the calculation with locomotive excesses is amply sufficient for all practical purposes, and he believes that, in view of the rapid increase of the weight of engines and

cars, the uniform load should be rather larger than generally taken, say 3 300 or 3 500 pounds per foot. The assumption of one particular engine with its actual load, which in a few years may be exceeded in weight by other engines on the same road, seems to render the assumption of actual loads for other cases than the calculation of the floor, of hip verticals, etc., rather a needless refinement, and in some cases quite as inaccurate as the assumption of a reasonable uniform load with locomotive excesses would be. Nevertheless, actual loads are used in many large offices, even in the calculation of double intersection trusses, and the following methods may therefore prove acceptable to some. The use of influence lines will be found to afford some aid in such cases.

Let us consider, for instance, the ordinary Whipple truss shown in Fig. 26, Plate III. We have seen that the lines of influence enable us to find: (1) the points where a single load must be to cause no stress in the bar considered; (2) the portion of the structure which must be covered by a uniform load in order to cause the maximum positive or negative stress; (3) the value of such stress, by means of areas; (4) for concentrated loads the loading causing maximum stress of either kind, by trial, or by deducing simple rules; (5) the value of such stress by the use of equation 13.

¶ In Fig. 26 we find the influence line for the chord piece a to be the broken line shown in Fig. 26A, the ordinates being indicated in the figure. There is some uncertainty regarding the loads on the joints 1 and 15. Thus, if we suppose the $\frac{1}{16}$ which goes to the left from joint 1 to go through the heavy system, the stress in a is $\frac{6}{16}\frac{p}{h}$, while if it goes through the light system it is $\frac{5}{16}\frac{p}{h}$, and if half on each it is $\frac{11}{32}\frac{p}{h}$. Overlooking this very small uncertainty, the apices of the influence line lie alternately on lines radiating from B and A. The numbers on the ordinate in the figure are, of course, to be multiplied by $\frac{p}{16\,h}$. Thus the stress in a for a load unity at 11 is $\frac{55}{16}\frac{p}{h}$. These ordinates may be easily found by any of the ordinary methods of calculation. Thus, of a load on 11, $\frac{11}{16}$ goes to the left abutment, and is the vertical component in the end post and all the diagonals of the light system to the left of a. The stress on the first and second upper chord pieces is therefore $\frac{11}{16}\frac{p}{h} + \frac{11}{16}\frac{2\,p}{h}$; and on the third and fourth $\frac{11}{16}\frac{p}{h} + \frac{11}{16}\frac{2\,p}{h} + \frac{11}{16}\frac{2\,p}{h} = \frac{55}{16}\frac{p}{h}$.

To find the loading causing the maximum stress in a, it is evident that the heaviest loads must be as near the point 10 as possible, with probably one load at that point. The skeleton of the truss and the influence line being drawn to scale, and the loads being plotted on tracing-cloth to the same scale, the loading causing maximum stress may be found by trial, placing the loads with one of the heaviest at 10, and finding $\Sigma P y$, then trying another load at 10 until the maximum is found. Or the following method may be used: first write on each panel the tangent of the angle which the influence line in that panel makes with the horizontal, calling it positive when the ordinate increases in going toward the left. This has been done in Fig. 26A on the line $A B$, the numbers to be multiplied by $\frac{p}{16 h}$ to obtain the actual tangents. Now place the tracing, with the plotted loads, over the truss, with one of the drivers just to the right of the point 10, and call G_1, G_2, G_3, etc., the total loads in the successive panels, commencing at the right. These loads can be very rapidly added up, and written above the numbers giving tangents. Then the stress will be increased by moving the system of loads to the left, provided

$$\Sigma (G \tan. \alpha) > 1 \qquad (23)$$

or in this case, if

$$6 G_1 + 6 G_2 + 3 G_3 + 9 G_4 + G_5 + 11 G_6 - G_7 + 13 G - 3 G_9 + 15 G_{10} - 5 G_{11} - 15 G_{12} - 7 G_{13} - 13 G_{14} - 9 G_{15} - 11 G_{16} > 1 \qquad (24).$$

Should this inequality hold, then we move the system to the left until the load which was previously just to the right of 10 is just to the left of the same point, and again apply the test. Should the inequality be now reversed, it would follow that the load in question must lie at the point 10 in order that the stress should be a maximum, and the stress itself would be obtained by equation 13. With a little practice this method will be found more rapid than appears at first sight, and it is of course as accurate as any method can be. All the bars of the truss may be treated in the same way, and it is not necessary to refer to them further. Fig. 26B, however, gives the influence line for the diagonal b, showing the general shape of all the lines for the diagonals.

Triangular trusses with two, three or more systems of web, and in fact all multiple systems, may be treated in the same way, and offer ground for no special remark.

X. Trusses with Curved Chords.

A. *Stresses in the Diagonals.*—Taking the most common case of a truss with curved chords, that shown in Fig. 27, Plate III, we shall first have to investigate the loading causing maximum stresses in diagonals. Considering the diagonal b, the ordinates to the influence line may best be obtained by the method of moments, the section being ss', and the origin of moments O, at a distance m to the left of the abutment A. The lever arm of the bar b is s, as shown, and the ordinates at C and D may be easily checked. If we have to do with a uniform load, we have now only to find the areas of the two triangles A_1 and A_2, and if p is the live and g the dead load per running foot, the maximum tension in b will be

$$(p + g) A_1 - g A_2 \tag{25}$$

and the minimum tension will be

$$g A_1 - (p + g) A_2 \tag{26}$$

and if the latter comes out negative it shows compression, or the necessity for a counter.

If we are dealing with a system of concentrated loads, the loading causing maximum stress may be found as follows: As the train approaches from the right, the stress will evidently increase until the first wheel reaches D, since, when there are no loads to the left of D, the stress is simply equal to $\frac{R_1 \, m}{s}$. In general, if M be the moment, about the point O, of all the loads and reactions to the left of D, the stress in the bar will be $S = \frac{M}{s}$.

Now if the first wheel be advanced beyond D, the reaction will be increased, and therefore its moment, but there will be a load on C, whose moment is to be subtracted, and both of these changes will take place uniformly as the load advances to the left. The question therefore is, as before, whether the stress will be greater with the first or with the second load at D. Let $R =$ the total load on the bridge when the first wheel is at D; then if the loads be all advanced a distance a_1, bringing the second wheel to D, the reaction will be increased by $\left(R \frac{a_1}{l} + d\right)$, and its moment therefore by $\left(R \frac{a_1}{l} + d\right) m$.

But we shall now have to subtract the moment of the load on C and all joints to the left. Let the distance $C O = n$, and if $a_1 > p$, let the distance from O to P_1 when P_2 is at D, be called r. Then we must

subtract $P_1 \frac{a_1}{p}.n$ or $P_1 r$, according as a_1 is less or greater than p. We shall therefore increase the stress by moving the load to the left, provided that

$$\left(R\frac{a_1}{l}+d\right)m > P_1\frac{a_1}{p}.\ n \quad (\text{when } a_1 < p) \tag{27}$$

$$\left(R\frac{a_1}{l}+d\right)m > P_1\ r \qquad (\text{when } a > p) \tag{28}$$

Since we can almost always neglect d, which is the reaction due to any load which may come on the span from the right during the operation, we have generally

$$R\frac{a_1}{l}m > P_1\frac{a_1}{p}n, \text{ or } R\frac{m}{l} > P_1\frac{n}{p} \text{ (when } a_1 < p) \tag{29}$$

$$R\frac{a_1}{l}\ m > P_1\ r, \text{ or } R\frac{m}{l} > P_1\frac{r}{a_1} \qquad (\text{when } a_1 > p) \tag{30}$$

Similar rules may easily be obtained with reference to the question of moving the third load to D, but they are not often necessary, and the method will be clear from the example below.

An example will show how easy these rules are of application. In Fig. 28, Plate III, let us consider the bar b. We first find m by the proportion

$$m + 40 : m + 60 :: 24 : 30$$

$$m = 40 \text{ feet.}$$

With the loads shown in Fig. 12 the stress in b will therefore be increased by moving up the second wheel to D, since

$$164\ \frac{7.5}{160}.\ 40 > 6.\ \frac{7.5}{20}.\ 80$$

$$41 > 24$$

Further, the stress would be still further increased by moving up the third wheel, provided

$$176\ \frac{4.5}{160}.\ 40 > 18.\frac{4.5}{20}.\ 80$$

$$447 > 72$$

hence the third load should not be moved up.

For counters, we may consider the diagonals sloping in the same direction from one end of the bridge to the other, thus calculating both main diagonals and counters. Thus in Fig. 29 we consider the truss to be as shown, and calculate all the diagonals. In the case of b', for instance, which is a counter, we proceed as follows:

The origin of moments is at O, the lever arm of b' is s', and the

influence line is drawn in Fig. 29A. In this case the stress will be greater when P_2 is at D than when P_1 is at D, provided

$$\left(R' \frac{a_1}{l} + d\right) m' > P_1 \frac{a_1}{p} n' \text{ (when } a_1 < p) \quad (27A)$$

$$\left(R' \frac{a_1}{l} + d\right) m' > P_1 r' \quad (\text{when } a_1 > p) \quad (28A)$$

or, neglecting d, when

$$R' \frac{m'}{l} > P_1 \frac{n'}{p} \quad (a_1 < p) \quad (29A)$$

$$R' \frac{m'}{l} > P_1 \frac{r'}{a_1} \quad (a_1 > p) \quad (30A)$$

just as before. R' being the resultant load on the span when P is at D, and n' and r' corresponding to n and r.

For finding the actual stress in a diagonal, the ordinary method is the following (Fig. 30): Find the shear V in the panel $C\ D$, and the stress S_1 in the upper chord. Then, since at the section $S\ S^1$ the sum of all the vertical forces must equal zero, we must have

$$V - S_1 \sin. \alpha_1 - S_3 \cos. \alpha_3 = o$$

$$S_3 = \frac{V - S_1 \sin. \alpha_1}{\cos. \alpha_3}$$

or, in other terms, we subtract from V the vertical component of S_1 to obtain the vertical component of S_3. The following method, however, will be found simpler and more expeditious. Let M_1 and M_2 be the moments, and h_1 and h_2 the heights of the truss at the points D and C respectively. Now, at the section $S\ S^1$ we must have the sum of the horizontal forces equal to zero; hence

$$S_1 \cos. \alpha_1 - S_2 - S_3 \sin. \alpha_3 = o \quad (31)$$

$$\text{but since } S_1 \cos. \alpha_1 = \frac{M_1}{h_1}, \text{ and } S_2 = \frac{M_2}{h_2}$$

$$\text{this reduces to } S_3 = \left(\frac{M_1}{h_1} - \frac{M_2}{h_2}\right) \text{cosec. } \alpha_3 \quad (32)$$

Take, for instance, the bar b in Fig. 27. The reaction at the left, for the loading causing maximum tension in b, is found to be 61.78 tons, and the load on C is 2.25 tons. Then we should have by the usual method

$$\frac{61.78 \times 60 - 2.25 \times 20}{30} = 122.06 = S_1 \cos. \alpha_1$$

$$122.06 \times \frac{6}{20} = 36.62 = S_1 \sin. \alpha_1$$

$$(61.78 - 2.25 - 36.62) \frac{31.24}{24} = 29.78 = S_3$$

By the last method we should have

$$\frac{61.78 \times 60 - 2.25 \times 20}{30} = 122.06 = \frac{M_1}{h_1}$$

$$\frac{61.78 \times 40}{24} = 102.97 = \frac{M_2}{h_2}$$

$$(122.06 - 102.97)\,\frac{31.24}{20} = 29.82 = S_2$$

B. *Stresses in the Verticals.*—These are best found by the simple method of moments. Thus in Fig. 30 the maximum stress in the vertical at C would occur for a load up to D, the origin of moments being at O. The loading causing maximum stress may be found just as has been explained, and the stress itself by taking moments about O.

C. *Stresses in the Chords.*—These also are found by the method of moments, finding the maximum moment at the proper joint opposite the chord piece in question, and dividing by the lever arm. Where there are counters it is necessary to make sure which diagon is in action in the panel in question, so that the correct origin of moments may be taken. For example, in Fig. 30, if the main diagonal $C^1\ D$ is in action, the origin of moments for the upper chord piece $C^1\ D^1$ is at D, and for the lower chord piece $C\ D$ it is at C^1, while if the counter is in action the origin for the upper chord is at C, and for the lower at D^1. In order to find maximum chord stress, it is therefore necessary to find the moments as follows:

{ Maximum M at B
Corresponding M at C (*i.e.*, for the same loading which causes the maximum at B)

{ Maximum at C
Corresponding M at B
Corresponding M at D

{ Maximum at D
Corresponding M at C
Corresponding M at E

{ Maximum M at E
Corresponding M at D
Corresponding M at F

Now, since in any panel, such as $C\ D$, the diagonal $C^1\ D$ is in action, provided $\frac{M_1}{h_1} > \frac{M_2}{h_2}$, as seen from equation (32), the above moments,

divided by the corresponding heights, enable us to determine easily the maximum stresses in the chords.

Suppose for instance that

$$A\left\{\begin{array}{l}\text{Maximum } M \text{ at } C = 480: \dfrac{M_c}{h_c} = 20\\[2ex] \text{Corresponding } M \text{ at } D = 570: \dfrac{M_d}{h_d} = 19\end{array}\right\}$$

$$B\left\{\begin{array}{l}\text{Maximum } M \text{ at } D = 630: \dfrac{M_d}{h_d} = 21\\[2ex] \text{Corresponding } M \text{ at } C = 432: \dfrac{M_c}{h_c} = 18\end{array}\right\}$$

From this it is evident that when the maximum M at C occurs, the diagonal $C D^1$ (counter) is in action, since $\dfrac{M_d}{h_d} < \dfrac{M_c}{h_c}$.

Hence in this case

$$\left\{\begin{array}{l}\text{Horizontal component of stress in upper chord} = 20\\ \text{Horizontal component of stress in lower chord} = 17\end{array}\right\}$$

When the maximum M at D occurs, the diagonal $C^1 D$ is in action, since $\dfrac{M_d}{h_d} > \dfrac{M_c}{h_c}$. Hence

$$\left\{\begin{array}{l}\text{Horizontal component of stress in upper chord} = 21\\ \text{Horizontal component of stress in lower chord} = 18\end{array}\right\}$$

Therefore we have

$$\left\{\begin{array}{l}\text{Horizontal component of maximum stress in upper chord} = 21\\ \text{Horizontal component of maximum stress in lower chord} = 19\end{array}\right\}$$

It will be easily seen that the stress in the upper chord is obtained from the greatest moment at either of the opposite joints, while that of the lower chord is obtained from the greatest of the two smaller moments in the sets A and B (19 and 18 in the above case).

XI. Cantilevers.

The stresses in cantilevers may be in many cases most easily found by the use of influence lines, and many of the principles which we have demonstrated find apt application in bridges of this class. We will therefore consider such structures somewhat in detail. As ordinarily built, for three spans, cantilevers are of two kinds: (1) those in which the cantilever extends over the center span into the side spans, support-

ing a simple girder on each end; and (2) those in which two cantilevers extend over the side spans and out into the center span, supporting a simple girder between them. Let us consider first the former kind, represented in Fig. 31, Plate IV. In both cases it may be premised, however, that the simple girders are to be treated exactly as any simple girder would be, so that we have here to do only with the cantilever portions.

(*a*) *Stresses in the Diagonals.*—In the truss represented in the figure, which has horizontal chords and a single web system, the web stresses are obtained directly from the shears, and the influence lines are drawn simply for the shears, or vertical components of the web stresses. Were the chords inclined, the influence lines would be drawn for the actual stresses in the bars, which could easily be found by the method of moments. Fig. 31A represents the influence line for the bar a. Any load on the span $E\,F$ is supported partly at E, and the supporting force at E increases directly with the distance of the load from F. A load unity at E causes a downward reaction of $\frac{5}{12}$ at C, and this is the shear in a. The ordinate at G is $\frac{8}{12}$, that at H, $\frac{4}{12}$, and that at B, $\frac{5}{12}$. Hence $B'\,H'$ and $G'\,E'$ are parallel straight lines, and the same is true of $a'\,B'$ and $E'\,F'$. For the maximum tension in a, the span $A\,C$ must be loaded, and also the span $C\,D$ from D to some point between H and G. For the maximum compression in a (or the maximum tension in the counter) the remainder of the bridge must be loaded. If the load is uniform we have simply to find the areas A_1, A_2, A_3, A_4, and we have, if g is the dead and p the live load per running foot,

$$\text{Maximum tension in } a = (g+p)\,(A_1 + A_3) - g\,(A_2 + A_4) \qquad (33)$$

$$\text{Maximum compression in } a = (g+p)\,(A_2 + A_4) - g\,(A_1 + A_3) \qquad (34)$$

If the actual concentrated loads are to be considered, for the maximum tension $D'\,K'$ must be heavily loaded, with the heaviest loads nearest to G, and $A'\,C'$ must be also fully fully loaded, while $C'\,K'$ must be entirely unloaded. I do not know that there is any uniform practice in this country with regard to cases like the above, where the loaded portions of a bridge are separated by portions entirely unloaded. In determining the maximum stresses in continuous girders, alternate spans must be loaded and unloaded, and in Austria it is fixed by law that the load may be divided into two parts, but no more. In the above case it will no doubt be sufficient to consider the actual loads to cover one portion ($A'\,C'$ or $K'\,D'$), and the uniform load the other, in

PLATE I
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Fig. 1.

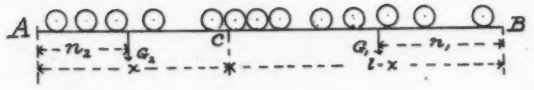


Fig. 2.

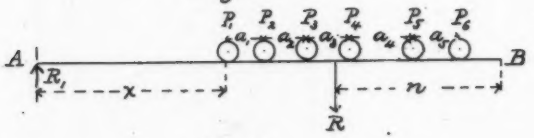


Fig. 3.

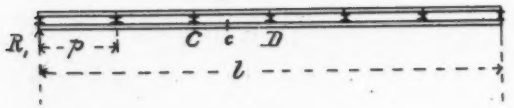


Fig. 4.

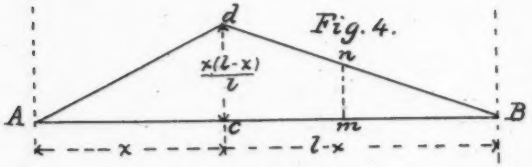


Fig. 5.

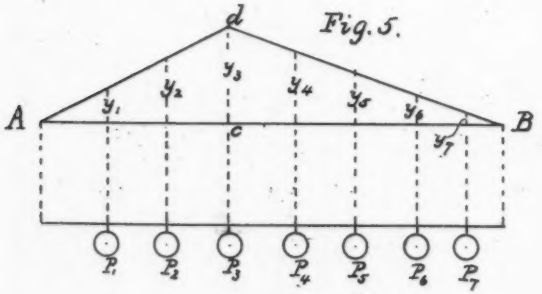


Fig. 6

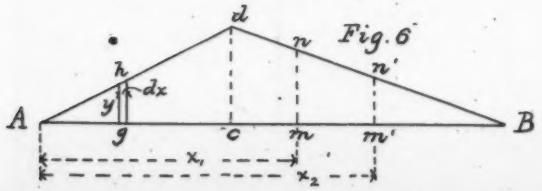


Fig. 7.

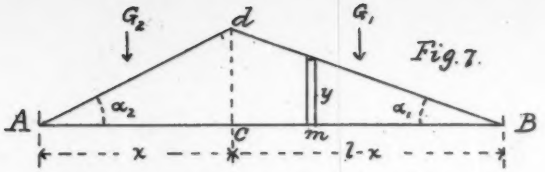


Fig. 8.

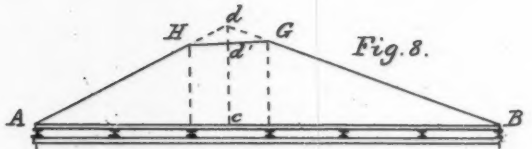
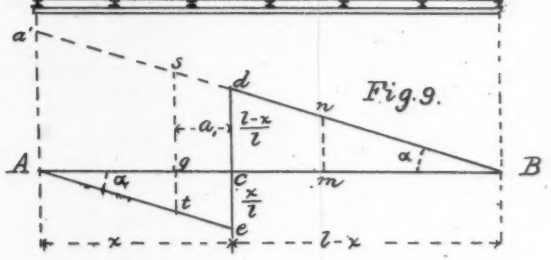


Fig. 9.



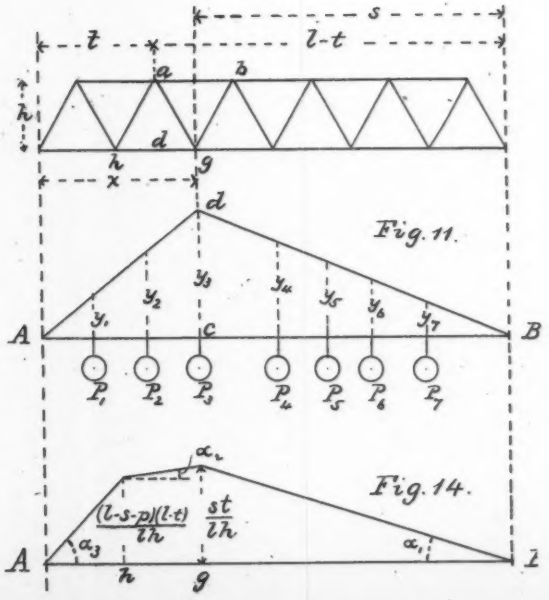
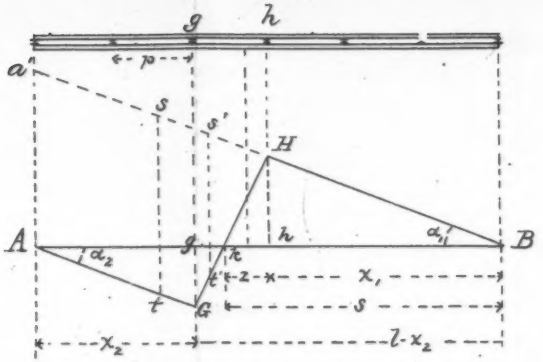


Fig. 10.



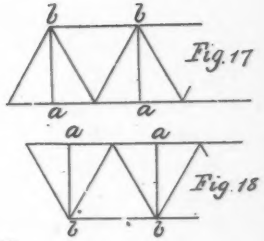
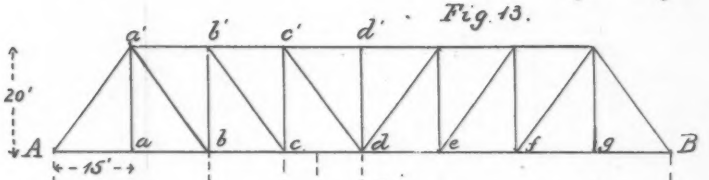
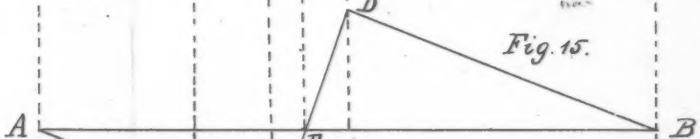
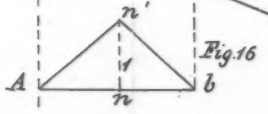
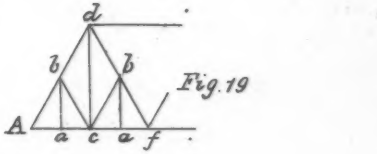


Fig. 13.









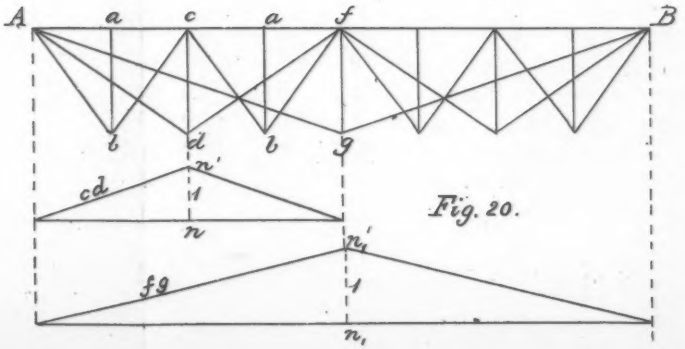


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Fig. 12.

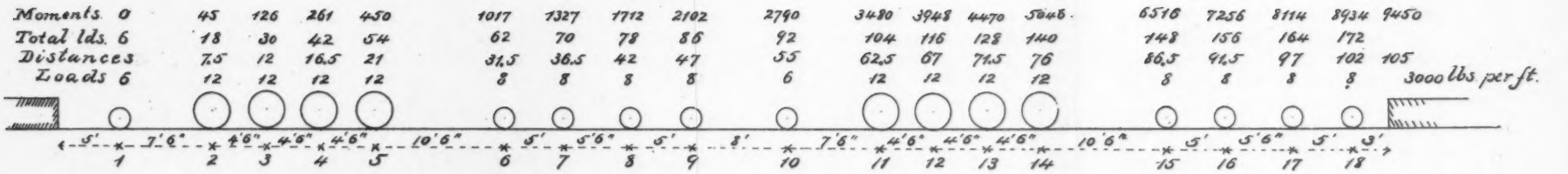

Fig. 21.

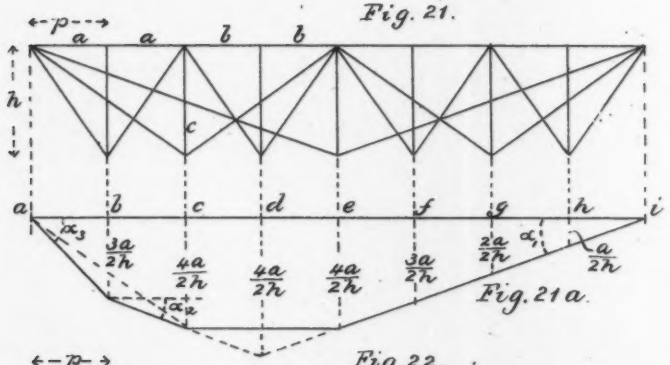

Fig. 21a.

Fig. 22.

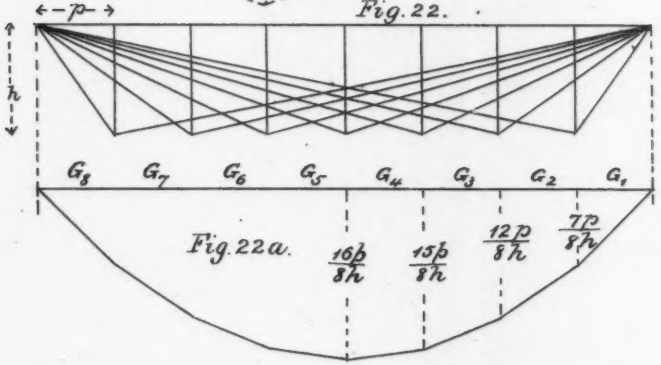

Fig. 22a.

Fig. 23.

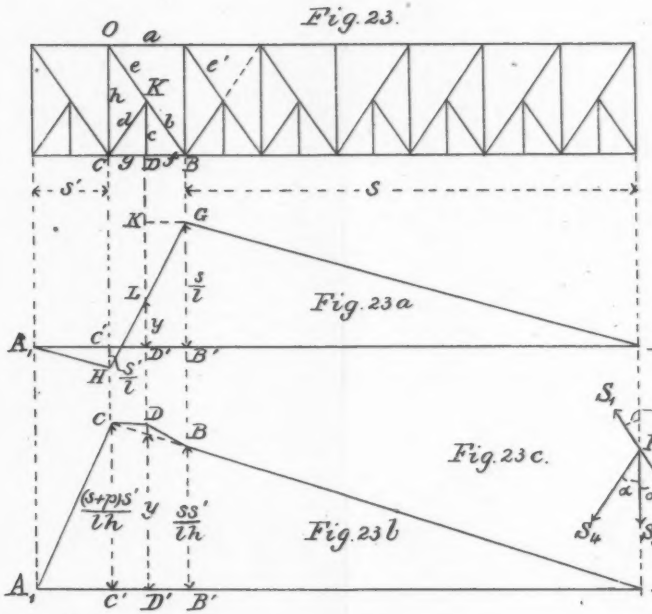

Fig. 23a.

Fig. 23b.

Fig. 23c.

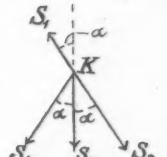

Fig. 24.

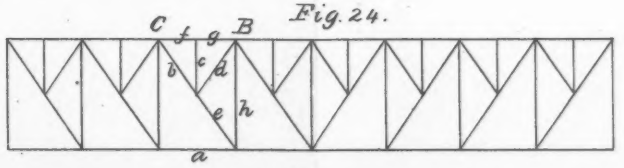

Fig. 25

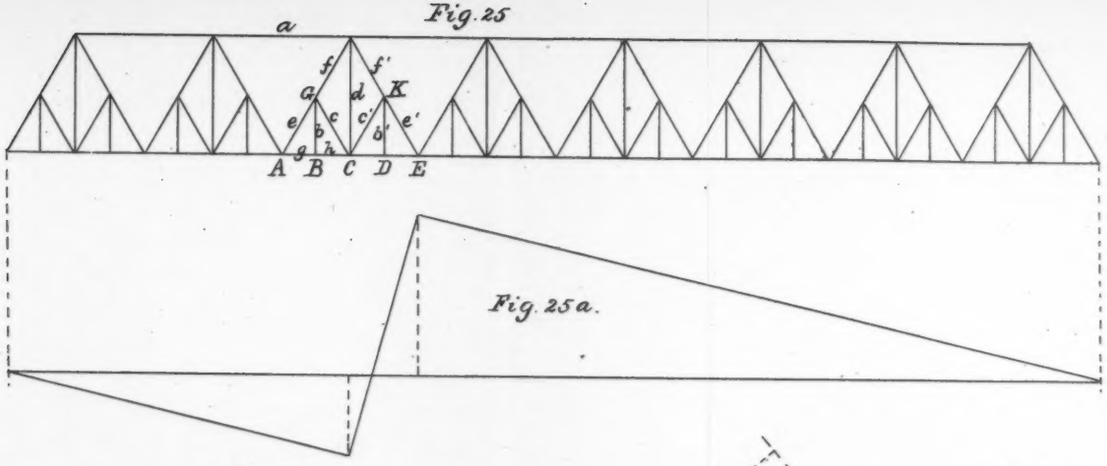

Fig. 25a.

Fig. 26

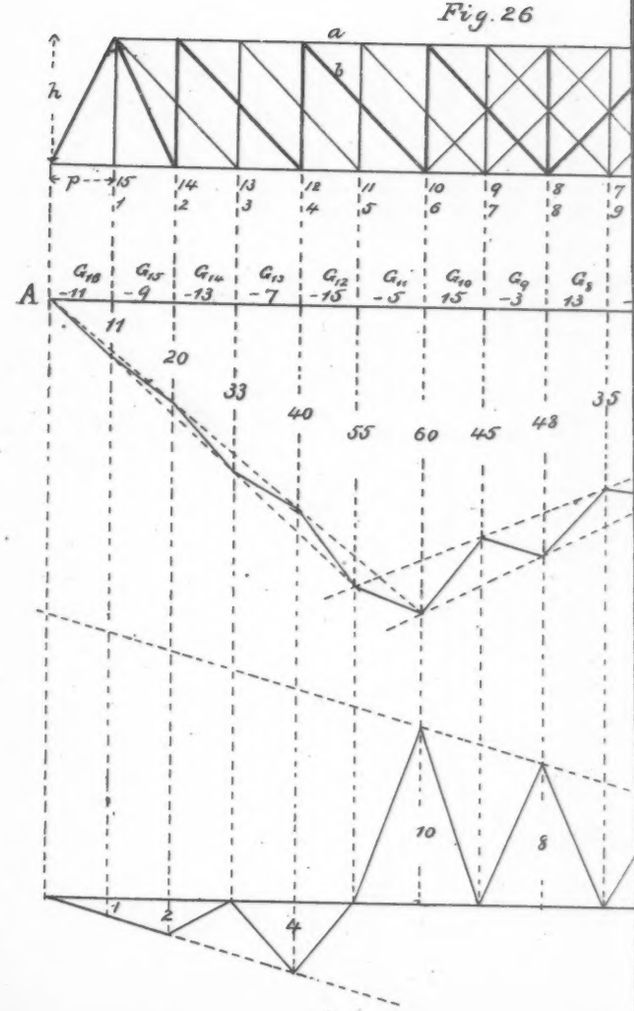

Fig. 28.

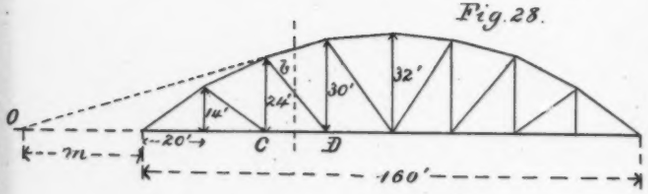

Fig. 27

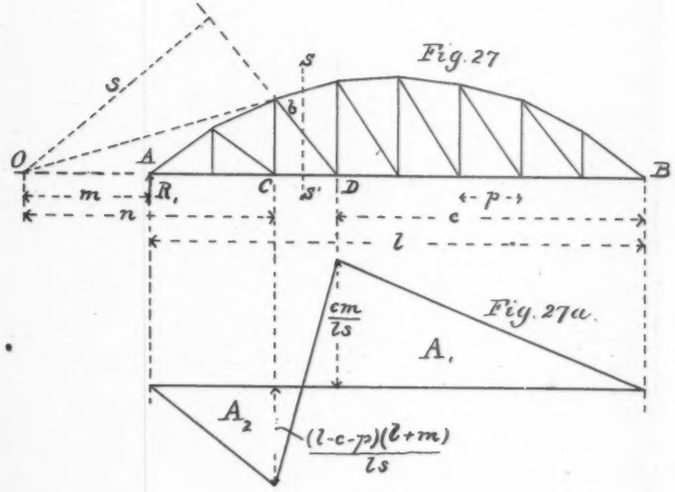

Fig. 27a.

Fig. 29.

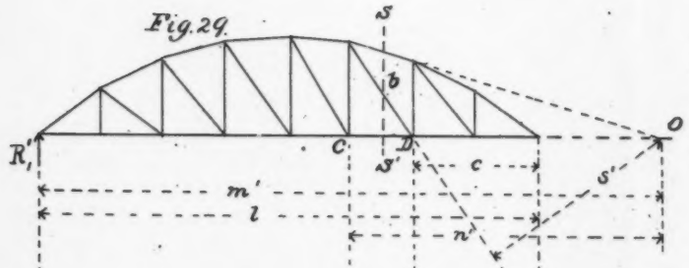

Fig. 29a.

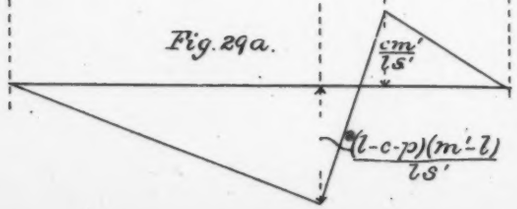

Fig. 30

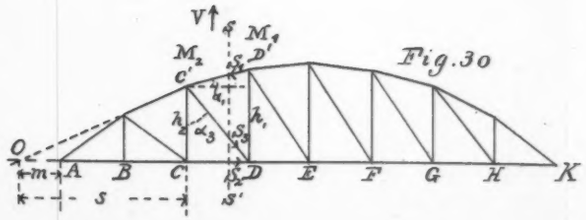

PLATE III
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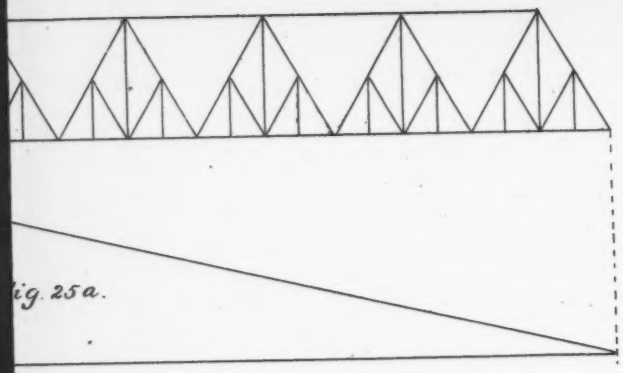
STRESSES IN BRIDGES.



ig. 25a.

Fig. 27

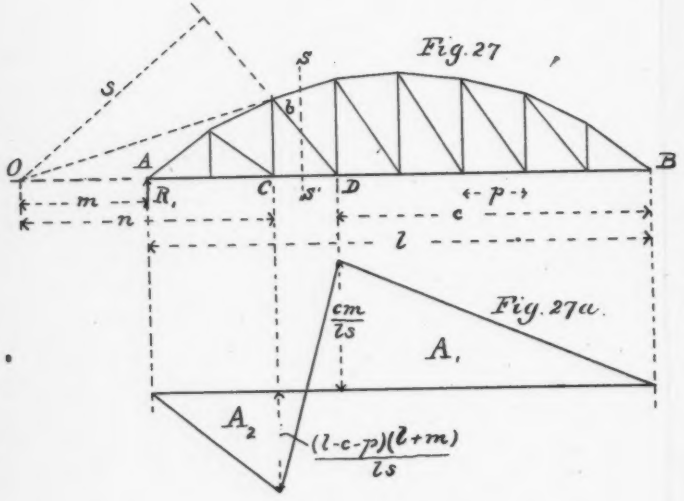


Fig. 27a.

Fig. 30

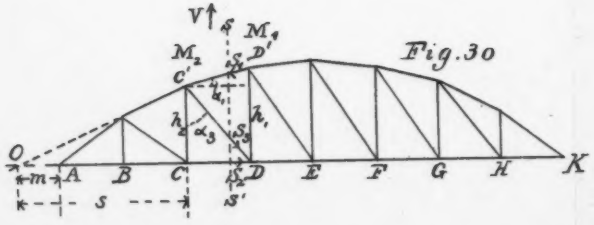


Fig. 26

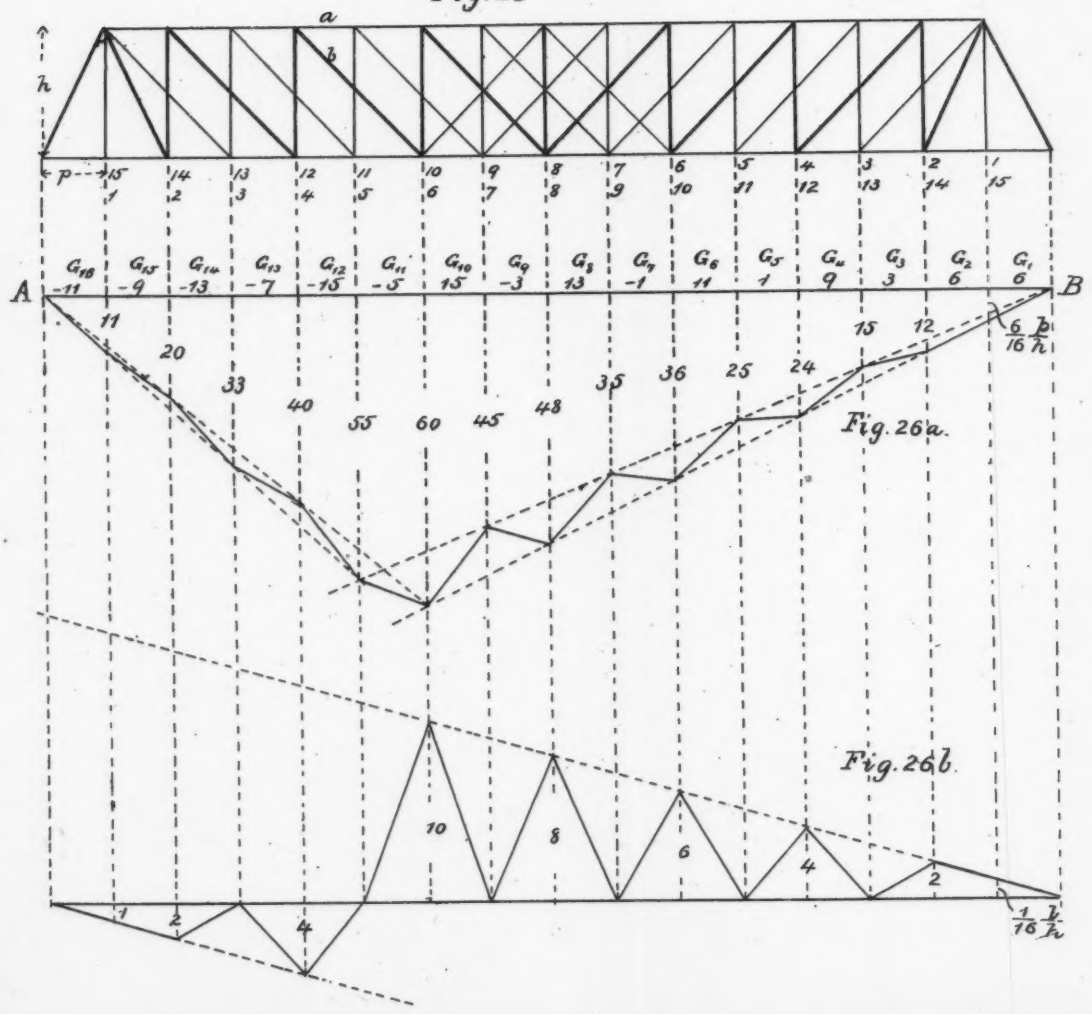


Fig. 26a.

Fig. 26b.

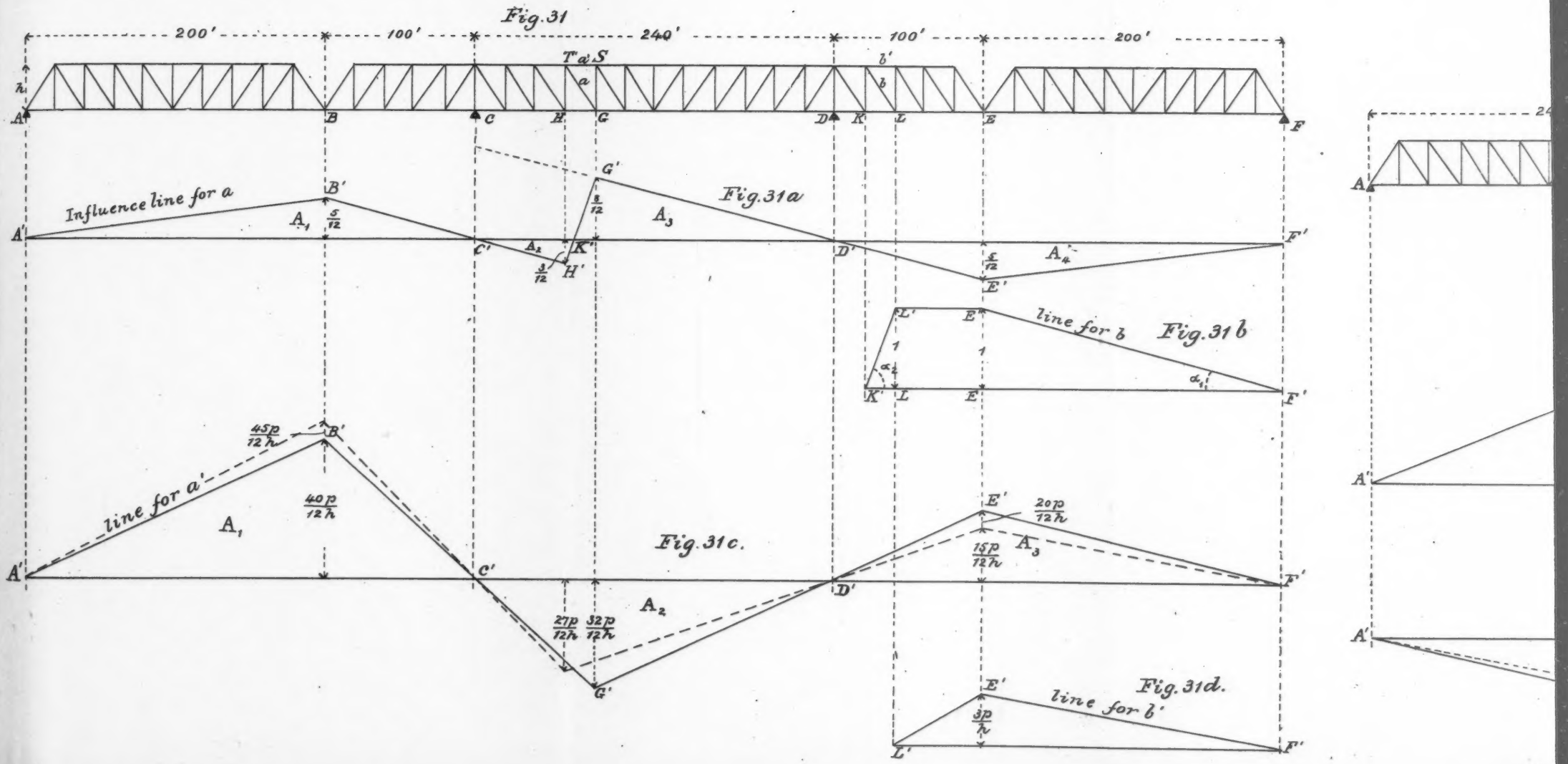


Fig. 31
200'
100'
240'
100'
200'
T a' S
a
b'
b
h
A
B
C
E
G
D
K
L
E
F
Influence line for a
B'
G'
Fig. 31a
A1
5/12
A2
A3
A4
A'
C'
K'
D'
F'
3/12
H'
E'
L'
E
line for b
Fig. 31b
1
1
α2
α1
K'
L
E
F'
45p/12h
B'
line for a'
40p/12h
A1
Fig. 31c.
A'
C'
D'
E'
20p/12h
15p/12h
A3
F'
A2
27p/12h
32p/12h
G'
Fig. 31d.
E'
line for b'
3p/h
L'
F'

PLATE IV
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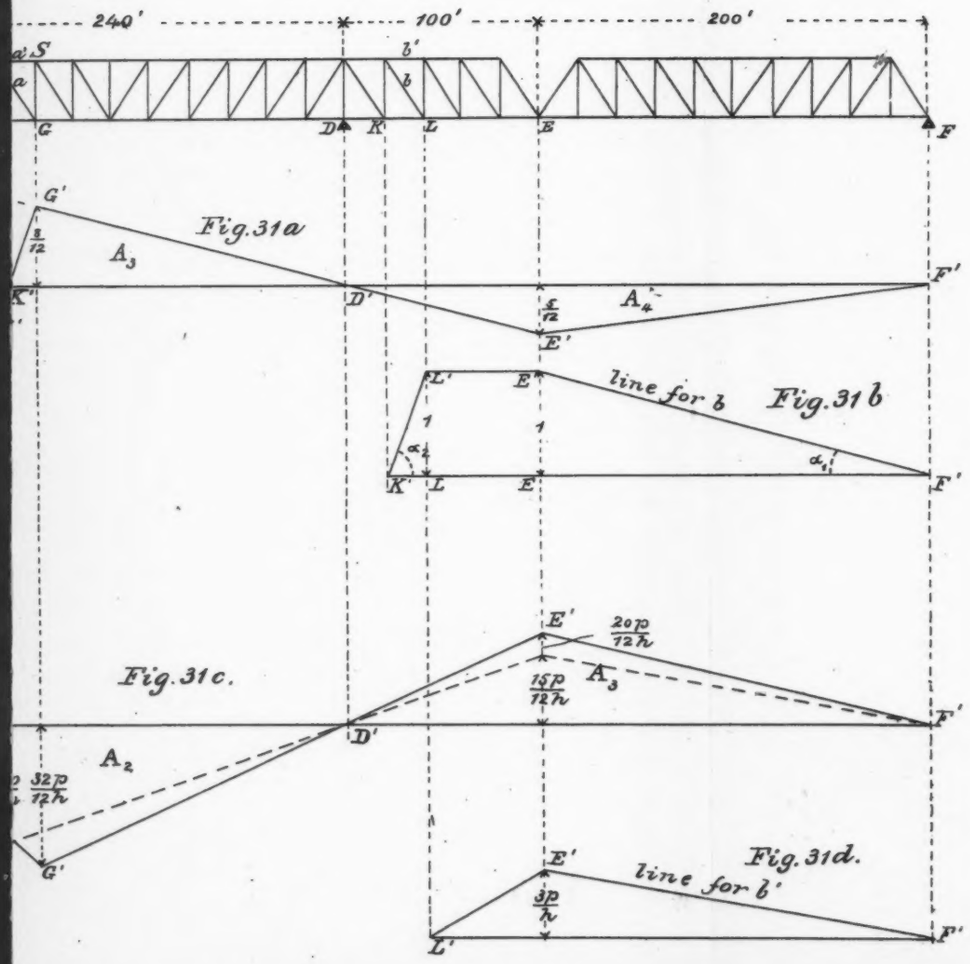
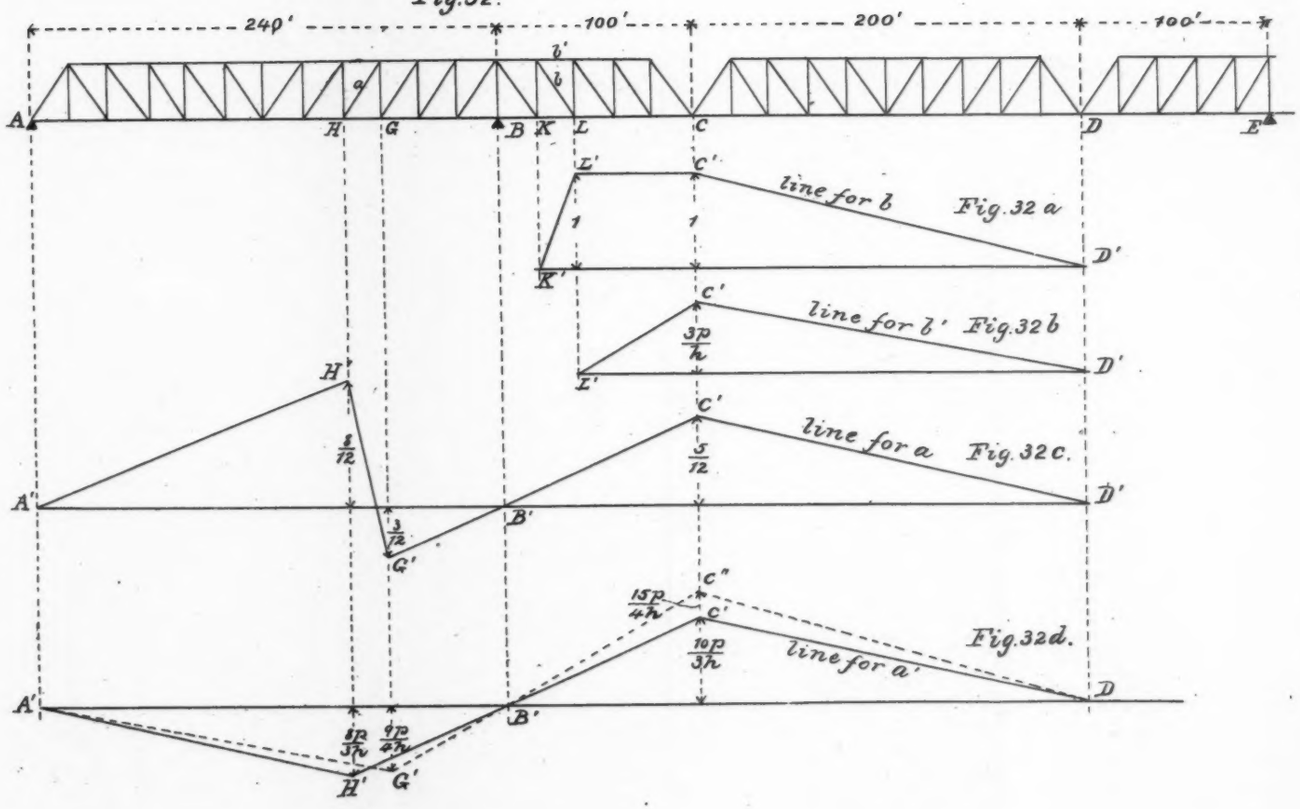


Fig. 32.



whichever way will give the greatest stress. If we therefore place the concentrated loads on $A\ C$, we must place them in the same way as to cause the maximum moment at B, and find that moment M. Then, according to what has been said on page 48, the stress in the bar a (in tons) due to the live load on $A'\ C'$ will be $\frac{1}{160}$ of the moment M (in foot tons), since for a load unity at B the moment at B would be $\frac{100 \times 200}{300} = 66\frac{2}{3}$, while the stress in a would be $\frac{5}{12}$, and therefore

$$\text{Stress} : M :: \frac{5}{12} : 66\tfrac{2}{3}$$

$$\text{Stress} = \frac{5}{12}\ \frac{3}{200}\ M = \frac{1}{160}\ M$$

Then the vertical component of the total maximum tension in a (in tons) would be (g and p being in tons per running foot)

$$V = g\ A_1 + (g+p)\ A_3 - g\ (A_2 + A_4) + \frac{1}{160}\ M \qquad (35)$$

We might then try the concentrated loads on $K'\ D'$ and the uniform load on $A'\ C'$, and see whether we obtain a greater stress.

The same method would be applicable to all the diagonals, counters and verticals in the span $C\ D$.

In the projecting portion $D\ E$, the influence line for the shear in the bar b is shown in Fig. 31B, the ordinates at E' and L' being unity. In order to find the loadings giving the maximum stress, a simple rule may be deduced; thus let G_1, G_2, G_3 be the total loads on $E'F$, $F\ L$ and $K\ L$ respectively. Then if the system of loads be moved a distance $d\ x$ to the left, the shear in b will be increased, provided

$$G_1 \text{ tan. } \alpha_1, > G_3 \text{ tan. } \alpha_3 \qquad (36)$$

or if

$$\frac{G_1}{E\ F} > \frac{G_3}{K\ L} \qquad (37)$$

The actual stress may be found by methods already explained.

(*b*) *Stresses in the Chords.*—Fig. 31C shows the influence line for the stress in the chord piece a'. Two influence lines may be plotted for each chord piece in panels where there are counters, one on the supposition that the counter is in action, the other on the supposition that the main diagonal is in action, for the origin of moments will be different in the two cases. In order to find the stress due to any given loading, we must first determine which diagonal is in action under that loading, and then use the corresponding line for the chords. The full line is for the case when the main diagonal $G\ T$ is in action, the dotted

line for the case when the counter $H S$ is in action. It is evident that for the maximum tension in a', the two outer spans must be completely loaded, and the center span entirely unloaded; further, that the heaviest loads must be in the span $A C$, and as near the point B as possible, since the ordinate there is larger than anywhere else. We must therefore place the concentrated loads on $A C$, and the uniform load on $D F$. On $A C$ we find the loading causing maximum moment at B, and that moment M, and we then determine, from the influence line for the diagonals, whether for this loading the main diagonal or the counter is in action. In this case it will probably be the main diagonal $F G$, and, supposing this to be the case, we use the full line and proceed as follows: The ordinate at B is $\frac{40\,p}{12\,h}$ but if the line $A'\ B'\ C'$ were for moments at B, the ordinate at the point would be $\frac{100 \times 200}{300} = 66\frac{2}{3}$, hence the stress in a' due to the load on $A\ C$ will be

$$\frac{M\frac{40\,p}{12\,h}}{66\frac{2}{3}} = M\frac{p}{20\,h}\ \text{(tons)} \qquad [M \text{ in foot tons}]$$

We therefore have for the maximum tension in a'

$$T \text{ maximum} = g\,A_1 - g\,A_2 + (g + p)\,A_3 + \frac{M\,p}{20\,h} \tag{38}$$

Similarly, if the line $C'\ G'\ D'$ were for moments, the ordinate at G' would be $\frac{32}{12}\,p$, and if M_1 is the maximum moment at G', the stress in a' is $\frac{M_1}{h}$. Supposing that for this loading the main diagonal $T G$ is in action, the maximum compression in a' is

$$C \text{ maximum} = g\,(A_2 - A_1 - A_3) + \frac{M_1}{h} \tag{39}$$

All the chord bars may be readily treated in this manner. Fig. 31A shows the influence line for the bar b', in the projecting portion $D E$, in which portion no counters are necessary. If this line were for moments at E, the ordinate at that point would be $\frac{30}{13}\,p$. Hence if M_2 is the maximum moment at E on a span $L F$, the maximum tension in b' will be

$$T' \text{ maximum} = M_2\frac{3\,p}{h} \cdot \frac{13}{30\,p} = 1.3\,\frac{M_2}{h} \tag{40}$$

There is no compression in this bar.

Let us finally consider the cantilever shown in Fig. 32, Plate IV. The projecting portion $B\,C$ is identical with the portion $D\,E$ of Fig. 31, and the influence lines in Figs. 32A and B are the same as in Figs. 31B and D. Fig. 32C shows the influence line for the diagonal a, the lines $A'\,H'$ and $G'\,C'$ being parallel. The concentrated loads would have to be placed on $B\,D$ in the same way as would cause the maximum moment at C, and on $A\,G$ in the same way as would cause the maximum shear in the panel $H\,G$. We may therefore, as before, try the concentrated loads over either of these portions and the uniform load over the other, and take whichever gives the maximum stress. Fig. 32D gives the two influence lines for the chord bar a', according as the main diagonal or the counter is in action, and the remarks on this matter made on page 49 apply here.

These illustrations are sufficient to show the application of this method to cantilevers, and will, I think, convince any one that when the actual loads are considered, it affords the easiest solution.

XII. Continuous Girders, etc.

As these structures are seldom built in this country except for drawbridges, it is not necessary to enter into a full and complete discussion of the calculation of stress by the use of influence lines. For concentrated loads, however, it will be found that the method is very convenient. Probably the simplest procedure is to find the reactions for a load unity at different points, and thus draw first the influence line for each reaction. This will enable the influence line for the stress in each bar to be constructed, after which the loading causing maximum stresses, and the stresses themselves, may be easily obtained.

Similar remarks will apply in the case of structures such as arches, with or without hinges. For the three-hinged arch, which is statically determined, this method is by far the easiest of application, even when uniform loads alone are considered. In this case it may be shown that the influence line for any bar consists of either three or four straight lines, so that the ordinates have to be found at only two, or at most three, points. The lines may be constructed rapidly, as the author knows by experience, and the areas are easily found. For concentrated loads there are some cases where it is difficult to see how the correct maximum stresses could be otherwise determined.

Were all the possible examples to be considered, the limits of this

paper would be unduly extended. Enough has been said, however, to bring to the attention of the profession in America this beautiful and fertile method.

XIII. Literature.

In closing, a list of the works and papers treating on this subject, so far as the writer is acquainted with them, is appended, that those who desire may investigate the matter more in detail. The list is arranged chronologically, with some brief notes.

1. Weyrauch. "Allgemeine Theorie und Berechnung der continuirlichen und einfachen Träger," pp. 50 and 101. 1873. (First suggestion of influence lines applied to continuous girders, but method not developed.)
2. Fränkel. "Über die ungünstigste Einstellung eines Systems von Einsellasten auf Fachwerkträger, mit Hülfe von Influenzcurven." *Der Civilingenieur*, 1876. (Method quite fully developed.)
3. Winkler. "Beitrag zur Theorie der Bogenträger." Zeitschrift des Architekten und Ingenieurs Vereins zu Hannover. 1879. (Some remarks on influence lines applied to arches.)
4. Melan. "Beitrag zur graphischen Behandlung der Fachwerksträger, mit Zugrundelegung des Principes der Influenzcurven." Zeitschrift des Architekten und Ingenieurs Vereins zu Hanover, 1880, p. 221. (Application to parallel and curved-chord bridges, with some interesting results.)
5. Stelzel. "Theorie einfacher, statische bestimmter Brückenträger." Wien, 1880. (Application to shears and moments on a simple beam.)
6. Müller—Breslau. "Theorie und Berechnung der eisernen Bogenbrücken." Berlin, 1880. (Application to arches.)
7. Winkler. "Theorie der gegliederten Balkenträger." Wien, 1881. Second edition. (First edition, 1875.)
8. "Handbuch der Ingenieur-wissenschaften." Edited by Schäffer & Sonne. Vol. 2, Chap. 10. Leipzig, 1882. (Illustrates influence curves for moments and shears on simple and continuous girders.)
9. Winkler. "Über die Belastungs-Gleichwerthe der Brückenträger." 1884. (Use of influence lines in determining the equivalent uniform load for bridges.)
10. Winkler. "Aeussere Kräfte der Balkenträger." Wien, 1886. Third edition. (Second edition, 1872.) (Application to shears and moments in simple and continuous girders.)
11. Müller—Breslau. "Die neueren Methoden der Festigkeitslehre." Leipsig, 1886. (Some applications to continuous girders, and to frames with superfluous bars.)
12. Levy. "La Statique graphique et ses applications aux Constructions." Paris, 1886. second edition. (Numerous applications of influence lines are given in this extensive work, the second edition of which reached the author's hands since the completion of this paper. Part II treats of applications to simple and continuous girders, and in the preface further applications to arches in the parts to follow are promised.)